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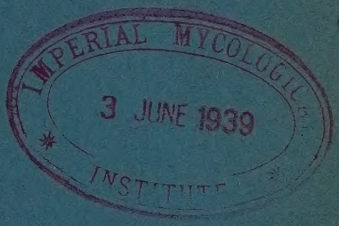

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MATHÉMATIQUES

SUR LA THÉORIE GÉNÉRALE DES ÉQUATIONS INTÉGRALES

Par N. GUNTHER, membre correspondant de l'Académie

1. Soient donnés un domaine (Ω) à l'étendue finie et un corps (C) de domaines appartenant à (Ω). Construisons une suite de réseaux d'intervalles

$$(R_1), (R_2), \ldots, (R_s), \ldots, \tag{1}$$

où (R_1) est un intervalle contenant (Ω) et où chaque réseau (R_s) suivant est obtenu en divisant les intervalles du précédent par des plans parallèles aux frontières de (R_1) de manière que chaque intervalle de la suite (1) appartienne au corps (C); en parlant d'un intervalle (i), nous envisageons seulement sa partie commune avec (Ω) [1].

Soit (E) un ensemble intégrable de mesure nulle. Désignons par (δ_s) l'ensemble des intervalles appartenant à (R_s) et contenant les points de (E). Posons $(\Omega_s) = (\Omega - \delta_s)$; soit (C_s) la portion de (C) faisant partie de (Ω_s). Supposons que la fonction moyenne $\varphi(\omega)$ est définie dans chaque (Ω_s), y est additive et à variation bornée, (C_s) étant son corps de continuité, mais n'est pas à variation bornée dans les domaines contenant les points de (E). La fonction $F(x)$ étant mesurable (B) et l'intégrale du second membre — celle de Stieltjes-Radon, nous posons, si la limite en question existe:

$$\int_{(\Omega)} \varphi(\omega) F(x) d\omega = \lim \int_{(\Omega_s)} \varphi(\omega) F(x) d\omega, \quad s \to \infty, \tag{2}$$

en disant que l'intégrale converge absolument, si l'intégrale

$$\int_{(\Omega)} V(\omega) F(x) d\omega, \tag{3}$$

$V(\omega)$ étant la variation de $\varphi(\omega)$, a un sens.

Choisissons pour fonction de mesure une fonction moyenne $u(\omega)$, à valeurs positives, vérifiant les conditions précédemment mentionnées. Nous dirons que

$$f(\omega) = \frac{1}{\omega} \int_{(\omega)} u(\omega) f(x) d\omega \tag{4}$$

est la moyenne mesurée de la fonction $f(x)$ mesurable (B); nous dirons que $f(x)$ est à carré sommable dans (Ω), si l'intégrale

$$\int_{(\Omega)} u(\omega) |f(x)|^2 d\omega \tag{5}$$

a un sens. En gardant pour les domaines (Ω_s) les définitions de l'intégrale de Hellinger, données dans mon Mémoire [1] et ma Note [2], convenons de dire que $\varphi(\omega)$ est de la classe (H), si les sommes

$$\sum_{k=1}^{k=n} \frac{|\varphi(\omega_k)|^2}{u(\omega_k)} \omega_k, \tag{6}$$

dans lesquelles $(\omega_1), \ldots, (\omega_n)$ sont des portions de (Ω_s), restent bornées par un nombre (H) indépendamment du choix des (ω_k) et du nombre s.

2. En envisageant l'équation

$$\varphi(x) = \lambda \int_{(\Omega)} k(\tau, x)\, \varphi(y)\, d\tau + f(x), \tag{1}$$

dans laquelle le noyau $k(\tau, x)$ n'est pas éventuellement à variation bornée dans (Ω), nous dirons qu'elle est de la classe (K), si les conditions suivantes sont satisfaites: a) comme fonction de (τ) $k(\tau, x)$ est défini dans chaque (Ω_s), a le corps (C_s) pour corps de continuité, est dans (Ω_s) à variation bornée, cette variation étant à carré sommable dans (Ω); b) si le domaine (τ) n'a pas de points communs avec (E), $k(\tau, x)$ est mesurable (B) et à carré sommable dans (Ω); c) le noyau $k(\tau, x)$ est hermitien [2]; d) si $K(\tau, x)$ est la variation moyenne de $k(\tau, x)$, on a, (x) appartenant à (Ω_s),

$$\int_{(\Omega_s)} K(\Omega_s, z)\, \Omega_s\, K(\xi, x)\, d\xi < B_s K(\Omega_s, x)\, \Omega_s, \tag{2}$$

où B_s est une constante, qui tend, éventuellement, vers l'infini avec s.

Les équations traitées dans le Mémoire de M. T. Carleman [3] sont de la classe (K), ainsi que l'équation de M. H. Weyl, exclue dans ledit Mémoire. Pour aborder les équations de la classe (K) nous traitons, en suivant M. Carleman, les équations

$$\varphi(x) = \lambda \int_{(\Omega_s)} k(\tau, x)\, \varphi(y)\, d\tau + f(x), \tag{3}$$

dont la théorie est esquissée dans ma Note [2]. On peut construire pour ces équations la fonction spectrale $\theta_s(\tau, \omega, m)$, dans laquelle (τ) et (ω) sont des domaines appartenant à (Ω_s) et (m) est un intervalle de l'axe réel du plan de la variable λ, dont les propriétés sont énumérées dans le paragraphe 9 de ma Note [2]. Parmi lesdites propriétés nous indiquerons l'égalité:

$$\int_{(m)} \frac{\theta_s(\tau, \omega, m)}{\mu}\, dm = \int_{(\Omega_s)} k(\tau, z)\, \theta_s(\xi, \omega, m)\, m\, d\xi, \tag{4}$$

qui a lieu pour chaque intervalle (m), y compris les intervalles contenant le point $\mu = 0$ et les intervalles infinis, et l'inégalité

$$\left| \int_{(\Omega_s)} h_1(x) \left(\int_{(\Omega_s)} h(y)\, \theta_s(\tau, \omega, m)\, m\, d\tau \right) d\omega \right| <$$

$$< \sqrt{\int_{(\Omega_s)} u(\omega)\, |h_1(x)|^2\, d\omega} \sqrt{\int_{(\Omega_s)} u(\omega)\, |h(x)|^2\, d\omega}. \tag{5}$$

3. La fonction $\theta_s(\tau, \omega, m)$ comme fonction de (m) est définie pour sle intervalles dont les extrémités n'appartiennent pas à un ensemble dénombrable (E_s). Soit (E_*) l'ensemble contenant les points de tous les (E_s). Formons les réseaux des intervalles

$$(r_1), (r_2), \ldots, (r_n), \ldots,$$

dont les extrémités n'appartiennent pas à (E_*). En formant les combinaisons des intervalles (τ_i), (ω_j), (m_k) faisant partie des réseaux $(1, 1)$ et $(1, 3)$, numérotons ces combinaisons dans un ordre quelconque. En considérant toutes lesdites combinaisons, on peut choisir une loi de succession des valeurs s_n de s de manière que la variable $\theta_{s_n}(\tau_i, \omega_j, m_k)$ possède une limite $\theta(\tau_i, \omega_j, m_k)$ pour chaque combinaison. En utilisant les inégalités (5, 2) on parvient à définir une fonction $\theta(\tau, \omega, m)$ pour tous les (τ) et (ω) du corps (C), n'ayant pas de points communs avec (E) et pour les intervalles (m), dont les extrémités n'appartiennent pas à un ensemble dénombrable (E_0). Le corps (C) est le corps de continuité de $\theta(\tau, \omega, m)$; la supposition qu'il existe plusieurs fonctions $\theta(\tau, \omega, m)$ n'est pas exclue.

4. On démontre pour chaque fonction $\theta(\tau, \omega, m)$ que, $h(x)$ et $g(x)$ étant des fonctions à carré sommable, l'intégrale

$$\theta(h, g, m) = \int_{(\omega)} g(x) \Big(\int_{(\omega)} h(y)\, \theta(\tau, \omega, m)\, d\tau \Big) d\omega \qquad (1)$$

a un sens; qu'on peut intervertir l'ordre de sommation dans (1); que

$$|\theta(h, g, m)|_m < \sqrt{\int_{(\omega)} u(\omega)\, |g(x)|^2\, d\omega}\ \sqrt{\int_{(\omega)} u(\omega)\, |h(x)|^2\, d\omega} \qquad (2)$$

et que lesdites conclusions subsistent même quand la mesure de (m) devient infinie; on peut même les généraliser en introduisant une fonction continue $\varphi(\mu)$ bornée dans l'intervalle $(-\infty, +\infty)$ en démontrant que l'intégrale

$$\int_{(\omega)} g(x) \Big(\int_{(\omega)} h(y) \Big(\int_{(-\infty)}^{(+\infty)} \theta(\tau, \omega, m)\, \varphi(\mu)\, dm \Big) d\tau \Big) d\omega \qquad (3)$$

a un sens et qu'on peut intervertir l'ordre de toutes les sommations; nous désignons par

$$\int_{(-\infty)}^{(+\infty)} \theta(h, g, m)\, \varphi(\mu)\, dm \qquad (4)$$

la limite de $\theta(h, g, m)_m$, où $(m) = (\mu', \mu'')$, pour $\mu' \to -\infty$, $\mu'' \to +\infty$. Enfin on a l'égalité

$$\int_{(m)} \frac{\theta(\tau, \omega, m)}{\mu}\, dm = \int_{(\omega)} k(\tau, z)\, \theta(\xi, \omega, m)\, m d\xi, \qquad (5)$$

qui subsiste même, quand (m) contient le point $\mu = 0$. On a

$$\int_{(\omega)} h(y) \Big[\int_{(m)} \frac{\theta(\tau, \omega, m)\, dm}{\mu} \Big] d\tau = \int_{(m)} \frac{\theta(h, \omega, m)}{\mu}\, dm; \qquad (6)$$

cependant le théorème, établi pour (1), ne subsiste pas, quand $\varphi(\mu) = \frac{1}{\mu}$ et l'intervalle (m) contient le point $\mu = 0$.

5. En désignant par (m_e) le lieu des points de l'axe réel dans le plan de λ, qui n'appartiennent pas à l'intervalle (m) de longueur finie, et en posant

$$\theta(\tau, \omega, m_e)\, m_e = \delta(\tau, \omega) - \theta(\tau, \omega, m)\, m, \tag{1}$$

où $\delta(\tau, x)$ est une fonction égale à zéro quand (x) est en dehors de (τ) et à $\frac{1}{\tau}$ dans le cas contraire, on démontre que l'égalité (5,4) subsiste pour l'intervalle (m_e) et, si $h(x)$ est à carré sommable, on a l'identité

$$h(\omega) = \int_{-\infty}^{+\infty} \Big(\int_{(\underline{\omega})} h(y)\, \theta(\tau, \omega, m)\, d\tau \Big) dm, \tag{2}$$

le second membre de (2) désignant l'intégrale suivant l'axe réel toute entière (4).

6. En convenant de dire que la fonction spectrale $\theta(\tau, \omega, m)$ est fermée, si

$$\delta(\tau, \omega) = \int_{(-\infty)}^{(+\infty)} \theta(\tau, \omega, m)\, dm, \tag{1}$$

on a, si $\theta(\tau, \omega, m)$ est fermée, pour chaque fonction $h(x)$ à carré sommable:

$$h(\omega) = \int_{(-\infty)}^{(+\infty)} \Big(\int_{(\underline{\omega})} h(y)\, \theta(\tau, \omega, m)\, d\tau \Big) dm. \tag{2}$$

Vu l'égalité (5,4), on démontre en partant de l'identité (2,5) que, même quand $\theta(\tau, \omega, m)$ n'est pas fermée,

$$\int_{(\underline{\omega})} u(\omega)\, k(\tau, x)\, h(x)\, d\omega = \int_{(-\infty)}^{(+\infty)} \frac{1}{\mu} \Big(\int_{(\underline{\omega})} h(x)\, \theta(\tau, \omega, m)\, d\omega \Big) dm;$$

$$k(\tau, \omega) = \int_{(-\infty)}^{(+\infty)} \frac{\theta(\tau, \omega, m)}{\mu}\, dm. \tag{3}$$

7. Convenons de dire que la fonction $h(x)$ est de la classe (U), si a) elle est à carré sommable, b) si l'intégrale

$$\int_{(\underline{\omega})} k(\tau, x)\, h(y)\, d\tau \tag{1}$$

a un sens et si c) $(e_n^{(s)})$ étant l'ensemble des points, dans lesquels

$$\Big| \int_{(\omega - \omega_s)} k(\tau, x)\, h(y)\, d\tau \Big| > n, \tag{2}$$

les ensembles $(e_n^{(\sigma)})$, $\sigma > s$, sont contenus dans $(e_n^{(s)})$ quand n est assez grand. La condition (c) est satisfaite, si (1) converge absolument et pour l'équation de M. H. Weyl.

On démontre que si la fonction $h(x)$ est de la classe (U)

$$\frac{1}{\omega} \int_{(\omega)} u(\omega) \Big(\int_{(\underline{\omega})} k(\tau, x)\, h(y)\, d\tau \Big) d\omega = \int_{(\underline{\omega})} k(\tau, \omega)\, h(y)\, d\tau = \int_{(\underline{\omega})} \overline{k(\omega, \tau)}\, h(y)\, d\tau. \tag{3}$$

8. Si la solution $\varphi(x)$ de l'équation (1, 2) appartient à la classe (U), on peut donner à cette équation la forme

$$\varphi(\omega) = \lambda \int_{(\Omega)} \overline{k(\omega, y)}\, \varphi(\tau)\, d\tau + f(\omega). \tag{1}$$

Si $f(x)$ est à carré sommable, la fonction

$$\varphi(\omega) = f(\omega) + \lambda \int_{(-\infty)}^{(+\infty)} \frac{1}{\mu - \lambda}\, \theta(f, \omega, m)\, dm \tag{2}$$

satisfait à l'équation (1). Si outre la fonction spectrale $\theta(\tau, \omega, m)$ il existe une autre $\theta_1(\tau, \omega, m)$ quelle que soit $f(x)$ à carré sommable, la fonction

$$\psi(\omega) = \lambda \int_{(-\infty)}^{(+\infty)} \frac{1}{\mu - \lambda}\, [\theta(f, \omega, m) - \theta_1(f, \omega, m)]\, dm \tag{3}$$

vérifie l'équation homogène

$$\psi(\omega) = \lambda \int_{(\Omega)} \overline{k(\omega, y)}\, \psi(\tau)\, d\tau. \tag{4}$$

Si $\theta(\tau, \omega, m)$ n'est pas fermée et

$$\varphi(\omega) = f(\omega) - \int_{(-\infty)}^{(+\infty)} \theta(f, \omega, m)\, dm, \tag{5}$$

on a pour chaque $f(x)$ à carré sommable

$$\int_{(\Omega)} \overline{k(\omega, y)}\, \varphi(\tau)\, d\tau = 0. \tag{6}$$

Si μ_k est un point de (E_0), si $(m) = (\mu', \mu'')$ contient μ_k et si

$$\theta(\tau, f, m) \, m \Rightarrow \varphi_k(\tau, f), \quad \mu' \to \mu_k, \quad \mu'' \to \mu_k, \tag{7}$$

la limite existant toujours, on a

$$\varphi_k(\tau, f) = \mu_k \int_{(\Omega)} k(\tau, z)\, \varphi_k(\xi, f)\, d\xi. \tag{8}$$

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MATHÉMATIQUES

SUR CERTAINES PROPRIÉTÉS DES *K*-ESPACES ÉTENDUS

Par A. G. PINSKER

(Présenté par S. N. Bernstein, de l'Académie, le 28. XII. 1938)

Dans une Note précédente (1) nous avons introduit la notion de *K*-espace étendu et indiqué une méthode pour le construire en partant d'un *K*-espace donné. Considérons, maintenant, certaines propriétés des *K*-espaces étendus qui les lient avec deux classes importantes de *K*-espaces, à savoir: avec les espaces réguliers (2) et les espaces contenant une unité (3).

Un espace s'appelle espace contenant une unité, si l'on peut indiquer dans cet espace un élément, nommé unité (et désigné 1), tel que $\inf(x, 1) > 0$ pour chaque élément $x > 0$. On a le théorème suivant:

Théorème 1. *Chaque K-espace étendu X est un espace contenant l'unité.*

Démonstration. Soit $E = \{x_\xi\}$ la suite transfinie de tous les éléments positifs de l'espace X. Désignons par E_1 l'ensemble des éléments positifs x satisfaisant à la condition $\inf(x, x_1) > 0$. Soit x_{ξ_2} le premier élément de la suite E n'appartenant pas à E_1; désignons par E_2 l'ensemble des éléments $x > 0$ pour lesquels $\inf(x, x_{\xi_2}) > 0$ et, en général, si les éléments x_{ξ_η} et les ensembles E_η sont déterminés pour tous les $\eta < \alpha$, désignons alors par x_{ξ_α} le premier élément de la suite E n'appartenant pas aux ensembles $\{E_\eta\}_{\eta<\alpha}$ et par E_α l'ensemble des éléments $x > 0$ pour lesquels $\inf(x, x_{\xi_\alpha}) > 0$.

Ainsi les suites transfinies des éléments $\{x_{\xi_\alpha}\}$ et les ensembles $\{E_\alpha\}$ qui leur correspondent seront déterminés. Évidemment

$$\inf(x_{\xi_{\alpha'}}, x_{\xi_{\alpha''}}) = 0, \text{ si } \alpha' \neq \alpha'' \qquad (1)$$

et pour chaque $x > 0$ il se trouvera α tel que

$$\inf(x, x_{\xi_\alpha}) > 0. \qquad (2)$$

Montrons que $x^* = \sup_\alpha \{x_{\xi_\alpha}\} < +\infty$. Dans ce but formons l'ensemble G composé de: 1) tous les éléments x_{ξ_α}, 2) les supremum's de tous les ensembles bornés possibles de ces éléments et 3) tous les éléments positifs dont chacun est plus petit qu'un élément de la première ou de la deuxième catégorie déjà indiquée.

L'ensemble G représente un complexe fini. En effet, si la suite $\{na_0\}$, $a_0 > 0$ $(n = 1, 2, \ldots)$ appartenait à G, alors pour un certain $\alpha = \alpha^*$ on aurait

$$\inf(a_0, x_{\xi_{\alpha^*}}) = a^* > 0.$$

Soit maintenant n un nombre entier arbitraire; alors il se trouvera un α tel, que $\inf(na^*, x_{\xi_\alpha}) > 0$ et, par conséquent, $\inf(a^*, x_{\xi_\alpha}) > 0$; si $\alpha \neq \alpha^*$, alors $\inf(x_{\xi_\alpha}, x_{\xi_{\alpha^*}}) > 0$, ce qui est contraire à (1); de là $\alpha = \alpha^*$ et

$$\inf(na^*, x_{\xi_{\alpha^*}}) = a_n^* > 0.$$

Il reste à démontrer que $na^* = a_n^*$; en effet, dans le cas contraire $na^* - a_n^* > 0$, $\inf(na^* - a_n^*, x_{\xi_{\alpha^*}}) = 0$ et selon (2) on a pour un certain α

$$0 < \inf(na^* - a_n^*, x_{\xi_\alpha}) \leqslant \inf(na^*, x_{\xi_\alpha}).$$

Nous en concluons, comme auparavant, que $\alpha = \alpha_0$, c'est-à-dire, $\inf(na^* - a_n^*, x_{\xi_{\alpha^*}}) > 0$, ce qui est impossible; ainsi $na^* = \inf(na^*, x_{\xi_{\alpha^*}})$ et $na^* < x_{\xi_{\alpha^*}}$ pour tous les n. La contradiction obtenue nous mène à la conclusion, que G est un complexe fini et $x^* < +\infty$. Si $x > 0$ est un élément arbitraire de l'espace X, alors pour un certain α

$$\inf(x, x^*) \geqslant \inf(x, x_{\xi_\alpha}) > 0.$$

L'élément x^* possède les propriétés d'une unité de l'espace, et X est un espace contenant une unité.

Pour les espaces réguliers la proposition suivante et importante est vraie (2): si $x_n \to x$, alors il existe un élément x_0, nommé régulateur de convergence, tel que quel que soit $\varepsilon > 0$, $|x_n - x| \leqslant \varepsilon x_0$, pour tout $n \geqslant N_\varepsilon$.

Une telle convergence «à régulateur» a lieu non seulement dans les espaces réguliers, à savoir:

Théorème 2. *La convergence dans K-espace étendu est une convergence à régulateur.*

Démonstration. Soit X un espace étendu, donc un espace contenant une unité, $x_n \to x$ et $\mathfrak{E} = \{e\}$ — l'ensemble des éléments unitaires de l'espace X, c'est-à-dire des éléments satisfaisant aux conditions: $0 \leqslant e \leqslant 1$ et $\inf(e, 1-e) = 0$, alors, quel que soit $\varepsilon > 0$,

$$|x_n - x| \leqslant \varepsilon(1 - e_n) + x_{e_n}^*, \text{ pour } n \geqslant N_\varepsilon,$$

où $e_n \to 0$ en décroissant,

$$x^* = \sup_n \{|x_n|\} \text{ et } x_{e_n}^* = \sup_m \{\inf[me_n, x^*]\}.$$

Posons $x_0 = \sum_{n=1}^{\infty} n(x^*)_{e_n - e_{n+1}} + 1$; $x_0 < +\infty$, comme les éléments $\{x_{e_n - e_{n+1}}^*\}$ satisfont à la condition: $\inf[x_{e_n - e_{n+1}}^*, x_{e_m - e_{m+1}}^*] = 0 \quad (m \neq n)$.

Pour chaque nombre entier k on trouve un $N_k > k$ tel que pour $n \geqslant N_k$

$$|x_n - x| \leqslant \frac{1}{k}(1 - e_n) + x_{e_n}^* \leqslant \frac{1}{k} 1 + \sum_{m=k}^{\infty} \frac{m}{k} x_{e_m - e_{m+1}}^* \leqslant$$

$$\leqslant \frac{1}{k} 1 + \frac{1}{k} \sum_{m=1}^{\infty} m x_{e_m - e_{m+1}}^* = \frac{1}{k} x_0.$$

Ainsi l'élément x_0 sera un régulateur de convergence de la suite $\{x_n\}$.

Théorème 3. *Soit $\{x_n\}$ une suite d'éléments de K-espace étendu X et $\lim_{n \to \infty} x_n = 0$, alors il existe une suite de nombres réels λ_n telle que*

$$\lim_{n\to\infty}\lambda_n=+\infty,\quad \lim_{n\to\infty}\lambda_n x_n=0.$$

Démonstration. D'après le théorème précédent il existe un élément x_0 — régulateur de convergence — tel que $|x_n|\leqslant\frac{1}{k^{2s}}x_0$ pour tout $n\geqslant n_s$ $(n_1<n_2<\ldots)$. Posons $\lambda_n=1$, pour $n<n_1$, et $\lambda_n=k^s$, pour $n_s\leqslant n<n_{s+1}$. Évidemment, $\lim\limits_{n\to\infty}\lambda_n=+\infty$, $\lambda_n|x_n|\leqslant\frac{1}{k^s}x_0$ et $\lim\limits_{n\to\infty}\lambda_n x_n=$ $=\lim\limits_{n\to\infty}\lambda_n|x_n|=0$.

On sait que le théorème démontré est vrai pour les espaces réguliers (²).

Théorème 4. *L'extension d'un K-espace régulier X sera régulier dans le cas et dans ce cas seulement où X est un espace contenant une unité.*

Démonstration. Soit X un espace régulier contenant une unité; montrons que son extension $\overline{X}$ sera aussi régulier. Il suffit de montrer que si $E_n\subset\overline{X}$, $\sup E_n=x_n$ et $\lim\limits_{n\to\infty}x_n=x$, il existe alors des ensembles finis $F_n\subset E_n$ tels que

$$\lim_{n\to\infty}[\sup F_n]=x.$$

Soit $x^*=\sup\limits_n[\sup E_n]$. Définissons pour chaque élément positif $x\in\overline{X}$ et pour chaque élément unitaire e l'élément

$$x_e=\sup_n\{\inf[ne,\ 1]\}.$$

Si $E\subset X$, $E=\{x\}$, alors l'ensemble $\{x_e\}$ sera désigné par le symbole E_e. Soit

$$e_m=e_m(x)=\sup_n\{\inf[n(x-m)_+,\ 1]\},\quad x_n^{(m)}=(x_n)_{1-e_m}\ \text{et}\ x_m=x_{1-e_m}.$$

Les ensembles $(E_n)_{1-e_m}$ se composent d'éléments bornés et ainsi appartiennent à l'espace X. Vu la régularité de X, il existe des ensembles finis $G_n^{(m)}\subset(E_u)_{1-e_m}$, $\sup G_n^{(m)}=y_n^{(m)}$ tels que $\lim\limits_{n\to\infty}y_n^{(m)}=x_m$ $(m=1,\ 2,\ \ldots)$.

Comme X est un espace contenant une unité, on peut indiquer alors pour chaque $\varepsilon_m>0$, $\varepsilon_m\to0$, un élément unitaire $e^{(m)}$ tel que pour tout $n\geqslant N_m$

$$|y_n^{(m)}-x_m|\leqslant\varepsilon_m(1-e^{(m)})+x^*_{e^{(m)}}\qquad(m=1,\ 2,\ \ldots).\qquad(*)$$

D'ailleurs, en tenant compte de la régularité de l'espace X, on peut supposer que pour des nombres convenablement choisis ε_m on a $e^{(m)}\to0$. En effet, soit $\delta_k\to0$ et $e_k^{(m)}$ des éléments unitaires satisfaisant aux conditions:

$$|y_n^{(m)}-x_m|\leqslant\delta_k(1-e_k^{(m)})+x^*_{e_k^{(m)}},\ \text{où}\ \lim_{k\to\infty}e_k^{(m)}=0\quad(m=1,\ 2,\ \ldots).$$

D'après la propriété connue des espaces réguliers (²) il existe une suite d'indices $\{k_m\}$ telle que $e_{k_m}^{(m)}\to0$ quand $m\to\infty$.

Il suffit maintenant de poser dans l'inégalité (*)

$$\varepsilon_m = \delta_{k_m} \quad \text{et} \quad e^{(m)} = e^{(m)}_{k_m}.$$

Soit $\{n_m\}$ une suite croissante d'indices et $n_m \geqslant N_m$, alors

$$|y^{(m)}_{n_m} - x_m| \leqslant \varepsilon_m (1 - e^{(m)}) + x^*_{e^{(m)}}. \qquad (**)$$

Considérons la suite des ensembles

$$\{G^{(k)}_n\}, \quad \text{où} \quad k = n_{m+1}, \quad \text{pour} \quad n_m < n \leqslant n_{m+1}.$$

Il est évident que l'inégalité (**) aura lieu pour chaque n satisfaisant à la condition $n_m < n \leqslant n_{m+1}$. Formons les ensembles $\{F_n\}$ en posant $x \in F_n$, si $x \in E_n$, et $x_{1-e_m} \in G^{(m)}_n$ (si plusieurs éléments satisfaisant à la deuxième condition appartiennent à E_n, nous prenons l'un d'eux). Les ensembles F_n sont finis et $F_n \subset E_n$.

Soit $\sup F_n = z_n$; on a l'inégalité suivante:

$$|x - z_n| \leqslant |x - x_m| + |x_m - y^{(m)}_n| + |y^{(m)}_n - z_n|,$$

où $n_m < n \leqslant n_{m+1}$. Si $n \to \infty$, on a aussi $m \to \infty$, $|x - x_m| \to 0$, par définition de l'élément x_m, $|x_m - y^{(m)}_n| \to 0$ d'après (**) et $|y^{(m)}_n - z_n| \to 0$ comme $z_n - y^{(m)}_n \leqslant x^*_{e_m}$. Il en résulte:

$$\lim_{n \to \infty} z_n = x.$$

L'extension de l'espace régulier contenant une unité est encore un espace régulier.

Soit maintenant X et son extension $\overline{X}$ réguliers. Montrons que X est un espace contenant une unité. Ainsi que dans la démonstration du théorème 1, formons les suites transfinies d'éléments $\mathfrak{E} = \{x_{\xi_\alpha}\}$ et des ensembles E_α qui leur correspondent. Si la suite $\mathfrak{E}$ est dénombrable, alors pour $\lambda_n \to 0$ convenablement choisis on a $x^* = \sum_{n=1}^{\infty} \lambda_n x_n < +\infty$ (la suite $\{x_n\}$ est formée de tous les éléments de la suite $\mathfrak{E}$), ce qui est facile à démontrer en tenant compte de la régularité de l'espace X, et l'on peut poser $x^* = 1$. Si la suite $\mathfrak{E}$ n'est pas dénombrable, alors

$$\sup \mathfrak{E} = x < +\infty$$

comme $\overline{X}$ est un espace étendu. Mais d'après la supposition $\overline{X}$ est régulier et d'après le théorème connu il existe un ensemble dénombrable $\mathfrak{E}' \subset \mathfrak{E}$ tel que $x' = \sup \mathfrak{E}' = x$. Si $x_{\xi_{\alpha_0}}$ n'appartient pas à $\mathfrak{E}'$, alors $\inf(x', x_{\xi_{\alpha_0}}) = 0$ tandis que $\inf(x, x^*_{\xi_{\alpha_0}}) = x_{\xi_{\alpha_0}} > 0$.

La contradiction obtenue démontre le théorème.

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MATHÉMATIQUES

SUR LES FONCTIONS PRESQUE PÉRIODIQUES APPARTENANT AU SENS STRICT À LA CLASSE *W*

Par B. LEVITAN et W. STEPANOV (STEPANOFF)

(Présenté par S. N. Bernstein, de l'Académie, le 9.XII.1938)

La définition des fonctions presque périodiques W [1] est la suivante: une fonction $F(x), -\infty < x < \infty$, sommable sur tout intervalle fini, est presque périodique W, si à tout nombre $\varepsilon > 0$ on peut assigner une longueur $L(\varepsilon)$ et un nombre $T(\varepsilon) > 0$ tel que pour un ensemble de nombres $\{\tau\}$ relativement dense par rapport à L* l'inégalité suivante a lieu pour $T > T(\varepsilon)$:

$$\frac{1}{T}\int_{\alpha}^{\alpha+T} |F(x+\tau) - F(x)|\,dx < \varepsilon, \quad -\infty < \alpha < \infty. \tag{1}$$

Des exemples de fonctions presque périodiques W sont fournies par les fonctions de la forme

$$F(x) = \varphi(x) + \psi(x), \tag{2}$$

$\varphi(x)$ étant presque périodique S** et $\psi(x)$ étant une fonction sommable sur tout intervalle fini satisfaisant à la condition

$$\lim_{T\to\infty} \sup_{\alpha} \frac{1}{T}\int_{\alpha}^{\alpha+T} |\psi(x)|\,dx = 0. \tag{3}$$

À notre connaissance, il n'existait pas jusqu'à présent d'exemple de fonction de la classe W proprement dite, c'est-à-dire d'une fonction satisfaisant à la condition (1), qu'il soit impossible de représenter dans

---

* Cela signifie que chaque intervalle de longueur L contient au moins un point de l'ensemble $\{\tau\}$.

** Une fonction $\varphi(x)$ est presque périodique S [2], si elle est sommable sur tout intervalle fini et si à tout $\varepsilon > 0$ correspond un $L(\varepsilon) > 0$ tel qu'il existe un ensemble $\{\tau\}$ relativement dense par rapport à L donnant lieu à l'inégalité

$$\int_{\alpha}^{\alpha+1} |\varphi(x+\tau) - \varphi(x)|\,dx < \varepsilon, \quad -\infty < \alpha < \infty.$$

la forme (2), en supposant $\psi(x) \neq 0$ sur un ensemble de mesure positive. Le but de la présente Note est de donner un tel exemple.

La fonction est de la forme

$$F(x) = \omega(f(x)),$$

$f(x)$ étant presque périodique réelle

$$0 < f(x) \leqslant 1, \ \inf_x f(x) = 0;$$

$\omega(u)$ est une fonction continue (non uniformément) pour $0 < u \leqslant 1$.

Précisons ces deux fonctions. Nous posons:

$$\omega(u) = \sin \frac{\pi}{2u}, \ 0 < u \leqslant 1.$$

La fonction $f(x)$ (périodique à la limite) est définie comme il suit.

Chaque nombre entier N peut être représenté d'une manière unique sous la forme

$$N = \alpha_0 + \alpha_1 \cdot 3 + \alpha_2 \cdot 3^2 + \ldots + \alpha_n \cdot 3^n, \ \alpha_i = 0, +1, -1. \tag{4}$$

Si pour un nombre N donné on a: $\alpha_k = 0$, $\alpha_{k'} \neq 0$ pour $k' < k$, nous dirons que l'intervalle $I_k = \left(N - \frac{1}{2}, N + \frac{1}{2}\right)$ est de rang k. Dans I_k nous posons:

$$f(x) = \frac{1}{k+2}, \qquad N - \frac{1}{4} \leqslant x \leqslant N + \frac{1}{4};$$

$$f(x) = 1 - 4\frac{k+1}{k+2}\left(x - N + \frac{1}{2}\right), \quad N - \frac{1}{2} \leqslant x \leqslant N - \frac{1}{4};$$

$$f(x) = 1 + 4\frac{k+1}{k+2}\left(x - N - \frac{1}{2}\right), \quad N + \frac{1}{4} \leqslant x \leqslant N + \frac{1}{2}.$$

La fonction $f(x)$ ainsi définie est uniformément continue (elle vérifie même la condition de Lipschitz avec la constante $= 4$). Elle est presque périodique: en effet, l'addition d'un nombre $m \cdot 3^{k+1}$ ($m = \pm 1, \pm 2, \ldots$.) à l'argument n'altère pas le rang d'un intervalle $I_{k'}$, $k' \leqslant k$; les intervalles de rang $> k$ sont transformés en ceux de rang $> k$. Par suite

$$f(x + m \cdot 3^{k+1}) = f(x), \ x \in I_{k'}, \ k' \leqslant k;$$

$$|f(x + m \cdot 3^{k+1}) - f(x)| \leqslant \frac{2}{k+3}, \ x \in I_l, \ l > k.$$

Ainsi tous les nombres $\tau = m \cdot 3^{k+1}$ sont des translations de f pour, tout $\varepsilon > \frac{2}{k+3}$. En outre, si $\varepsilon < \frac{1}{(k+2)(k+3)}$, toutes les translations $\tau(\varepsilon)$ sont contenues dans les intervalles $\left(m \cdot 3^{k+1} - \frac{\varepsilon}{4}, m \cdot 3^{k+1} + \frac{\varepsilon}{4}\right)$.

Examinons maintenant les propriétés de la fonction

$$F(x) = \sin \frac{\pi}{2f(x)}. \tag{5}$$

Dans les $I_{k'}$, $k' \leqslant k$, nous avons: $F(x + m \cdot 3^{k+1}) = F(x)$.

Soit maintenant $\tau = m \cdot 3^{k+1}$, $m \equiv \pm 1 \pmod 3$, alors on a la représentation:

$$\tau = \pm 3^{k+1} + \beta_{k+2} \cdot 3^{k+2} + \beta_{k+3} \cdot 3^{+3} + \ldots \pm 3^s. \tag{6}$$

Il existe un ensemble relativement dense d'intervalles de rang $k+1$, I'_{k+1}, dont les nombres N correspondants ont dans leurs développements (4) $\alpha_{k+1}=0$, $\alpha_{k+2}+\beta_{k+2}=0$, alors pour $x\in I'_{k+1}$, $x+\tau$ appartient à un I_{k+2}, pour $N-\frac{1}{4}\leqslant x\leqslant N+\frac{1}{4}$ il suit

$$\left|F(x+\tau)-F(x)\right|=\left|\sin\frac{\pi(k+3)}{2}-\sin\frac{\pi(k+4)}{2}\right|=1. \qquad (7)$$

Donc, aucune translation de la forme (6) de $f(x)$ n'est pas une telle pour $F(x)$; il en est de même pour tout nombre de la forme $\tau\pm\delta$ pour $\delta<\frac{1}{8}$, l'égalité (7) ayant lieu dans l'intervalle $\left(N-\frac{1}{8}, N+\frac{1}{8}\right)$; or, nous avons vu que toutes les translations de f sont de la forme $\tau\pm\delta$ avec $\delta<\frac{\varepsilon}{4}$. Par suite $F(x)$ n'a point pour ε suffisamment petit de translations au sens de H. Bohr, ni au sens S.

Il est facile de vérifier que $F(x)$ appartient à la classe W. En effet chaque segment de longueur 3^{k+1} contient les intervalles $I_{k'}$, $k'\geqslant k$, en nombre 2^k. Par suite

$$\frac{1}{T}\int_{\alpha}^{\alpha+T}|F(x+m\cdot 3^{k+1})-F(x)|\,dx\leqslant\frac{2}{T}\left(\frac{T}{3^{k+1}}+1\right)2^k=\left(\frac{2}{3}\right)^{k+1}+\frac{2^{k+1}}{T}.$$

La dernière expression sera inférieure à ε, si l'on choisit d'abord k de manière que $\left(\frac{2}{3}\right)^{k+1}<\frac{\varepsilon}{2}$ (alors $L(\varepsilon)=3^{k+1}$) et ensuite T de manière que $\frac{2^{k+1}}{T}<\frac{\varepsilon}{2}$, donc $T(\varepsilon)=\frac{2^{k+2}}{\varepsilon}$.

La fonction $F(x)$ appartient aussi à une classe considérée récemment par l'un des auteurs (3), contenant les fonctions presque périodiques de H. Bohr et caractérisée par les conditions suivantes:

I. Pour tout intervalle $(-N, N)$ et pour tout $\varepsilon>0$ il existe un ensemble relativement dense de nombres $\{\tau\}$ tels que $|F(x+\tau)-F(x)|<\varepsilon$ pour $-N\leqslant x\leqslant N$. II. Si, N étant donné, τ' correspond à ε' et τ'' correspond à ε'', alors $\tau'+\tau''$ correspond à $\delta(\varepsilon',\varepsilon'')$, δ tendant vers zéro avec $\varepsilon', \varepsilon''\to 0$

$F(x)$ vérifie ces conditions, comme toute fonction de la forme $\sin\frac{\pi}{2f(x)}$, où $f(x)$ est presque périodique, $f(x)>0$, $\inf_x f(x)=0$.

En effet on a:

$$\left|F(x+\tau)-F(x)\right|\leqslant 2\left|\sin\frac{\pi(f(x+\tau)-f(x))}{2f(x+\tau)\cdot f(x)}\right|.$$

Soit $f(x)\geqslant k>0$ pour $-N\leqslant x\leqslant N$; si τ (ainsi que τ') est une translation de $f(x)$ relative à $\varepsilon<\frac{k}{4}$, on obtient: $|F(x+\tau)-F(x)|<\frac{2\pi\varepsilon}{k^2}$ et $|F(x+\tau+\tau')-F(x)|<\frac{4\pi\varepsilon}{k^2}$, ce qui démontre les propriétés I et II.

Il est démontré (3) qu'une fonction de la classe considérée, vérifiant en outre les conditions:

$$a(\lambda)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T}F(x)\,e^{i\lambda x}\,dx$$

existe pour tout λ réel,

$$\overline{\lim_{T\to\infty}} \frac{1}{2T} \int_{-T} |F(x)|^2 dx < \infty,$$

est définie d'une manière univoque par sa série de Fourier, c'est-à-dire qu'une telle fonction supposée continue est identiquement nulle, si tous ses coefficients de Fourier s'annulent. Or, notre fonction $F(x)$ appartenant à la classe W vérifie bien ces deux conditions supplémentaires.

Maintenant nous pouvons démontrer que $F(x)$ ne peut pas être représentée sous la forme (2). Supposons en effet que

$$F(x) = \varphi(x) + \psi(x), \tag{2}$$

φ et $\psi \in L_1$, φ étant presque périodique S, et ψ vérifiant (3).

Il suit de l'identité (2):

$$\int_x^{x+h} F(\alpha)\, d\alpha = \int_x^{x+h} \varphi(\alpha)\, d\alpha + \int_x^{x+h} \psi(\alpha)\, d\alpha$$

ou bien

$$F_1(x) = \varphi_1(x) + \psi_1(x).$$

La fonction $F_1(x)$ appartient à la même classe que $F(x)$; φ_1 est presque périodique au sens de Bohr, donc la différence $\psi_1 = F_1 - \varphi_1$ appartient à la classe des fonctions définies d'une manière univoque par leurs séries de Fourier; or, cette serie est nulle, vu la propriété (3) de $\psi(x)$, donc $\psi_1(x) \equiv 0$, ou bien

$$\int_x^{x+h} F(\alpha)\, d\alpha = \int_x^{x+h} \varphi(\alpha)\, d\alpha,$$

par conséquent, $F(x) - \varphi(x)$ est une fonction périodique de période h; mais en vertu de la condition (3) elle est nulle presque partout, ce qu'il fallait démontrer.

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MATHÉMATIQUES

SUR L'ÉQUIVALENCE PAR RAPPORT À LA CONSTRUCTION DU COMPLEXE *MB* ET DU COMPLEXE *E*

Par N. NESTOROVITSCH

(Présenté par S. N. Bernstein, de l'Académie, le 28.XII.1938)

§ 1. Un grand nombre de lieux géométriques, employés dans la résolution des problèmes de construction dans l'espace de Lobatschevsky, correspond identiquement aux mêmes lieux géométriques dans l'espace d'Euclide. Mais les lieux géométriques des points, équidistants de la droite, se distinguent radicalement des parallèles d'Euclide, présentant une courbe nommée équidistante (hypercycle).

En rapport avec cette particularité de l'espace de Lobatschevsky il semblait * que pour la résolution des problèmes de construction, qui conduisent aux lieux géométriques indiqués, il est nécessaire, en dehors du compas et des la règle, d'avoir un instrument spécial nommé «hypercompas» pour tracer l'équidistante.

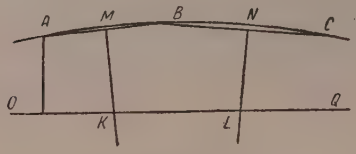

Fig. 1.

En résolvant les problèmes du système mentionné avec l'aide du complexe élargi: compas-règle-hypercompas, nommé dans la suite, pour abréger, «complexe *MB*» **, il a été mis en lumière, que si l'on est parvenu à la solution à l'aide du complexe *MB*, on parvient à trouver chaque fois la solution de ces problèmes à l'aide du complexe classique *E* (Euclide—compas, règle). Ceci suggéra l'idée de l'existence d'un théorème général au sujet de l'équivalence par rapport à la construction du complexe *MB* et de celui d'*E*. Les recherches suivantes ont montré que ce théorème est juste. Pour démontrer ce théorème, on suit les étapes que voici:

1) On examine tous les moyens possibles de donner l'équidistante. Ces moyens sont:

a) à l'aide de l'axe *OQ* et de la distance *h*;

b) à l'aide de l'axe *OQ* et de l'un des points *M* de cette courbe;

---

* Par exemple, la résolution des problèmes liés avec la surface, ou des problèmes contenant une ou plusieurs hauteurs.

** Les initiales du professeur D. D. Mordoukhay-Boltovskoy, qui introduisit ce complexe dans la géométrie de construction de l'espace de L. (1).

c) à l'aide de trois points A, B, C, appartenant à une branche de la courbe;

d) à l'aide de trois points A_1, B, C_1, dont deux points A_1 et C_1 appartiennent à une branche et le point B à une autre branche;

e) graphiquement par le segment de l'arc AC de l'une des branches.

On démontre ensuite que tous les moyens de donner l'équidistante dans le cas du complexe E peuvent être réduits à la considération de l'axe et de la distance.

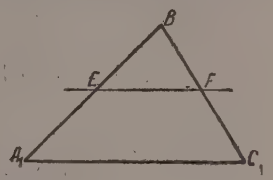

Fig. 2.

Voilà pourquoi dans la suite nous ne considérons que le seul moyen a) de donner l'équidistante, celui qui correspond au moyen habituel de donner la circonférence par la position du centre O et de la dimension du rayon r.

2) On démontre que toutes les constructions principales, étant accomplies à l'aide du complexe MB, peuvent être accomplies à l'aide du complexe E.

§ 3. Tous les cas possibles de l'application du hypercompas dans les problèmes de construction conduisent à la solution de l'un des problèmes suivants:

1) La recherche du point d'intersection de la droite et de l'équidistante.

2) La recherche du point d'intersection d'une circonférence et de l'équidistante.

3) La recherche du point d'intersection de deux équidistantes.

4) Au problème de tracer l'équidistante.

En ce qui concerne le problème 4), c'est juste que l'équidistante, comme lieu géométrique continu, ne peut être tracée qu'avec un instrument spécial, comme les autres lieux géométriques continus: par exemple, la circonférence dans les constructions de Steiner ou la droite — dans les constructions de Mascheroni.

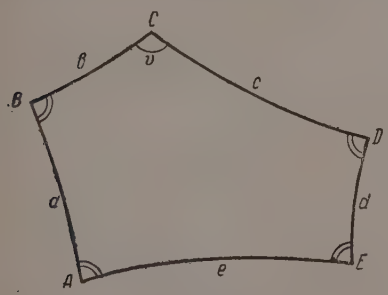

Fig. 3.

Mais de ce qui suit on peut conclure que n'importe quel nombre de points séparés de l'équidistante peut être construit à l'aide du complexe E.

§ 4. Dans la résolution du problème 1) il faut distinguer 3 cas:

a) la droite donnée rencontre l'axe de l'équidistante;

b) elle est parallèle à l'axe de l'équidistante;

c) la droite donnée et l'axe de l'équidistante sont surparallèles.

Le cas a) se réduit à la construction de l'hypoténuse d'un rectangle $\triangle$ d'après la cathète h (= distance) et l'angle opposé α (inclinaison de la droite donnée A_iB_i à l'axe de l'équidistante) et se résout par le complexe E.

En changeant la position de la droite A_iB_i nous pouvons construire n'importe quel nombre de points de l'équidistante.

Le cas b) conduit à la construction $\underline{/_2\backslash}$ d'après $(a_2, c_2\ \mu_2)$ [2] près de $\mu_2 = \Pi(a_2)$.

Le cas c) conduit à un autre cas particulier de la construction $\underline{/_2\backslash}$ d'après $(a_2, c_2\mu_2)$. Ces deux constructions se réalisent par le complexe $\underline{E}$ [2].

§ 5. La résolution du problème 2) conduit à la construction $\underline{/_2\backslash}$ d'après $(a_2b_2c_2)$ et se réalise de même par le complexe E [2].

§ 6. Dans la résolution du problème (3) il faut distinguer de nouveau 3 cas:

a) les axes des équidistantes données se rencontrent;

b) les axes sont surparallèles;

c) les axes sont parallèles.

Le cas a) conduit à la construction $\overline{\vee}$ d'après $(b^{(2)}, c^{(2)}, \lambda^{(2)})$.

Les cas b) conduit à la construction $\underset{1}{\wedge}$ d'après $(b_1 c_1 e_1)$. Ces deux constructions se réalisent par le complexe E (²).

Le cas c) conduit à la construction du segment c, qui se définit par la formule: $\operatorname{sh} c = \frac{\operatorname{sh} h_1 + \operatorname{sh} h_2}{\operatorname{ch} h_2}$, où h_1 et h_2 sont les distances des équidistantes données.

Comme sh du segment c est exprimé à l'aide d'une fraction rationnelle à coefficients rationnels en sh et ch des segments donnés h_1 et h_2, la valeur c en vertu des recherches du professeur D. D. Mordoukhay-Boltovskoy (¹) peut être construite par le complexe E. Toutes les constructions nécessaires peuvent être indiquées (¹).

Après la recherche du segment c la résolution se réduit à la résolution du problème 1 (§ 4).

§ 7. Déductions. L'emploi du hypercompas dans la résolution des problèmes de construction du second ordre dans l'espace de L. n'est pas nécessaire et son rôle peut être réduit:

1) à tracer les branches de l'équidistante comme lieu géométrique continu, puisque n'importe quel nombre de points isolés de l'équidistante peut être construit à l'aide du complexe E (§ 4);

2) à simplifier les constructions réalisées par le complexe E, comme il suit de la comparaison des évaluations géométriques des solutions obtenues par le complexe MB et par le complexe E [au sens de Lemoine (³)]. Donc, le rôle du hypercompas dans les constructions de la géométrie de L. est pareil à celui de la règle dans les constructions de Mascheroni ou celui du compas dans les constructions de Steiner. Il est partiellement pareil au rôle de l'équerre dans les constructions à l'aide du complexe E (sur le plan E) parce qu'en réalité on emploie toujours l'équerre dans les constructions géométriques à l'aide du complexe E pour les simplifier, bien que l'application de l'équerre n'est pas nécessaire.

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Rostov sur Don.

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MATHEMATIK

ZUR SPEKTRALTHEORIE MAXIMALER OPERATOREN

Von A. PLESSNER

(Vorgelegt von I. M. Vinogradow, Mitglied d. Akademie, d. 9.I.1939)

Es sei H ein unitärer (komplexer Hilbertscher) Raum, der auch nichtseparabel sein kann. Das skalare Produkt zweier Elemente f und g aus H bezeichnen wir mit (f, g), die Norm eines Elementes h mit $\| h \| = \sqrt{(h, h)}$; A sei ein maximaler hermitescher Operator, A^* — sein adjungierter Operator. Die Punkte derjenigen Halbebene, die keine komplexe Eigenwerte des Operators A sind, bezeichnen wir mit λ; die Punkte der anderen Halbebene, wo also die Resolvente $R_\mu = (A - \mu E)^{-1}$ existiert, mit μ und mit ν unterschiedslos λ oder μ, d. h. ν—eine beliebige komplexe Zahl, für welche $\operatorname{Im} \nu \neq 0$. Weiter setzen wir $R_\lambda = R_\lambda^*$. Hiermit ist R_ν für jedes ν definiert; diese Operatorenschar R_ν nennen wir die Quasiresolvente von A.

Hilfssatz 1. *Es gilt*

$$(R_\nu h, g) \to 0 \quad (\nu R_\nu h, f) \to 0$$

wenn $|\nu| \to \infty$, $|\operatorname{Im} \nu| > \vartheta > 0$, *$h$ und g beliebig aus H, f aus dem Definitionsbereich von A genommen wird.*

Mit funktiontheoretischen Mitteln weist man ferner nach Hilfssatz 2 und Satz I (unter Benutzung von Hilfssatz 1).

Hilfssatz 2. *Ist Δ ein ganz im Endlichen gelegenes Intervall, so gilt:*

$$\left| \int_\Delta (R_\nu h, g)\, d\sigma \right| = O\left(\lg \frac{1}{|\tau|} \right); \quad \nu = \sigma + i\tau.$$

Satz I. *Wird* $P(\sigma, \tau) = \frac{\operatorname{sgn} \tau}{2\pi i} (R_\nu - R_{\bar{\nu}})$ *gesetzt, so gilt:*

$$\int_{-\infty}^{\infty} P(\sigma, \tau)\, h \cdot d\sigma = h. \tag{1}$$

Vermittels $(E(\Delta) h, h) = \lim_{\tau \to 0} \int_\Delta (P(\sigma, \tau) h, h)\, d\sigma$ [Δ — Stetigkeitsintervall der Grenzfunktion $(E(\Delta) h, h)$] führen wir die Spektralfunktion $E(\Delta)$ als

additive Intervallfunktion ein und erhalten mit ihrer Hilfe Integraldarstellungen für die Quasiresolvente und den Operator A.

Satz II. *Für die Quasiresolvente R_ν gilt die Integraldarstellung*

$$R_\nu h = \int_{-\infty}^{\infty} \frac{dE(\Delta_\xi)\, h}{\xi - \nu}. \tag{2}$$

Satz III. *Für sämtliche f, die zum Definitionsbereich von A gehören, gilt die Integraldarstellung:*

$$Af = \int_{-\infty}^{\infty} \xi dE(\Delta_\xi) f. \tag{3}$$

f gehört dann und nur dann dem Definitionsbereich von A an, wenn das Integral $\int_{-\infty}^{\infty} \xi^2 d(E(\Delta_\xi) f, f)$ endlich bleibt und es ist:

$$(Af, Af) = \int_{-\infty}^{\infty} \xi^2 d(E(\Delta_\xi) f, f).$$

Satz IV. *Für die Spektralfunktion $E(\Delta)$ besteht ausser den evidenten Eigenschaften:* 1) $(E(\Delta)h, h) \geqslant 0$, 2) $E(\Delta) = E(\Delta_1) + E(\Delta_2)$, *wenn* $\Delta = \Delta_1 + \Delta_2$, *noch die weitere der Vollständigkeit:* 3) $E(\Delta)h \to h$ (*stark*, $\| h - E(\Delta) h \| \to 0$) *wenn* $\Delta = (a, b)$ *und* $a \to -\infty$, $b \to \infty$.

Die Spektralfunktion $E(\Delta)$ besitzt im Fall eines maximalen aber nicht hypermaximalen Operators nicht die Orthogonalitätseigenschaft; wir führen deshalb als Abweichung von der Orthogonalität den Operator $D(\delta, \Delta) = E(\delta\Delta) - E(\Delta)E(\delta)$ und nennen ihn die Diskrepanz des maximalen Operators oder der zugehörigen Spektralfunktion. Bekanntlich verschwindet die Diskrepanz dann und nur dann identisch, wenn A hypermaximal ist.

Satz V. *Fixiert man das Intervall Δ bzw. δ, so ist $(D(\delta, \Delta)h, h)$ eine additive totalstetige Intervallfunktion in δ bzw. Δ, sodass*

$$(D(\delta, \Delta)h, h) = \int_\delta \Phi(\xi, \Delta; h)\, d\xi = \int_\Delta \Psi(\delta, \eta; h)\, d\eta, \tag{4}$$

wobei $\Phi(\sigma, \Delta; h)$ und $\Psi(\delta, \sigma; h)$ Grenzfunktionen analytischer Funktionen $\Phi(\lambda, \Delta; h)$ und $\Psi(\delta, \mu; h)$ in der λ- und μ-Halbebene sind; diese analytischen Funktionen besitzen überdies einen beschränkten Mittelwert; d. h. ($\lambda = \sigma + i\tau$, $\mu = \sigma - i\tau$)

$$\int_{-\infty}^{\infty} |\Phi(\lambda, \Delta; h)|\, d\sigma < \varkappa(\Delta, h); \quad \int_{-\infty}^{\infty} |\Psi(\delta, \mu; h)|\, d\sigma < \varkappa(\delta, h). \tag{5}$$

Satz VI. *Besitzt eine Intervallfunktion $E(\Delta)$ die Eigenschaften* 1) *bis* 3) *aus Satz* IV, *und hat ihre Diskrepanz $E(\delta\Delta) - E(\Delta)E(\delta)$ die in Satz* V *angegebene Eigenschaften, so ist $E(\Delta)$ Spektralfunktion eines maximalen Operators.*

Eine wichtige Eigenschaft der Diskrepanz sprechen wir in Satz VII aus.

Satz VII. *Ist $G(\sigma)$ eine beschränkte messbare Funktion, so hat man:*

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\xi)\,\overline{G(\eta)}\,d_\xi d_\eta\,(D(\delta_\xi,\Delta_\eta)\,h,h) = \int_{-\infty}^{\infty} G(\xi)\,d_\xi \int_{-\infty}^{\infty} \overline{G(\eta)}\,\Psi(\delta_\xi,\eta;h)\,d\eta =$$

$$= \int_{-\infty}^{\infty} \overline{G(\eta)}\,d_\eta \int_{-\infty}^{\infty} G(\xi)\,\Phi(\xi,\Delta_\eta;h)\,d\xi \geqslant 0. \qquad (6)$$

Es sei nun $\mu_0 = -\varepsilon i$ ($\varepsilon = \operatorname{sgn}\operatorname{Im}\lambda$) und T der Eigenraum von A^* für den Eigenwert $\mu_0 = -\varepsilon i$, d. h. die Gesamtheit der Elemente g, sodass $A^*g + \varepsilon i g = 0$. Den Eigenraum T können wir uns von Elementen g_α aufgespannt denken, die normiert und zueinander orthogonal sind, d. h. $(g_\alpha, g_\beta) = \delta_{\alpha\beta}$ (hierbei kann α eine nicht abzählbare Menge durchlaufen). Die Elemente $E(\Delta)\,g_\alpha$ spannen bei veränderlichem Δ einen Teilraum H_α auf, der invariant inbezug auf A ist. Die invarianten Teilräume H_α und H_β sind zueinander orthogonal. Die Gesamtheit der Elemente aus H, die zu sämtlichen H_α orthogonal sind, bilden wieder einen invarianten Teilraum H_0. Man hat somit die Zerlegung des Grundraumes H in invariante zueinander orthogonale Teilräume

$$H = H_0 + \sum_\alpha H_\alpha; \qquad (7)$$

jedes Element h aus H wird also zerlegt

$$h = h_0 + \sum h_\alpha;\ h_0 \subset H_0;\ h_\alpha \subset H_\alpha, \qquad (7')$$

wobei jetzt h_α nur für höchstens abzählbar viele α von Null verschieden sein kann. Wir bezeichnen mit A_α bzw. A_0 den Operator A, wenn er nur auf dem Teilraum H_α bzw. H_0 betrachtet wird. Diese Teiloperatoren sind wieder hermiteisch und maximal, A_0 sogar hypermaximal. Für A_α^* besteht der Eigenraum T_α für den Eigenwert $\mu_0 = -\varepsilon i$ aus den Elementen cg_α [1].

Wir konstruieren nun einen Operator K, der sich isometrisch äquivalent mit jedem der Operatoren A_α erweisen wird und in gewisser Hinsicht die kanonische Darstellung der A_α gibt. Wir betrachten hierbei die Gesamtheit der analytischen Funktionen $F(\lambda)$ in der λ-Halbebene für die $\int_{-\infty}^{\infty} \frac{|F(\lambda)|^2}{|\lambda+\varepsilon i|^2}\,d\sigma < \infty$ ($\lambda = \sigma + i\tau$). Es sei $F(\sigma)$ die Grenzfunktion von $F(\lambda)$ für $\tau \to 0$. Die Gesamtheit der Funktionen $F(\sigma)$ wird ein unitärer Raum M, wenn wir das skalare Produkt zweier Elemente $F(\sigma)$ und $G(\sigma)$ aus dieser Gesamtheit durch

$$(F,G) = \int_{-\infty}^{\infty} \frac{F(\sigma)\overline{G(\sigma)}}{1+\sigma^2}\,d\sigma \qquad (8)$$

definieren. Der Operator K sei nun definiert durch

$$KF(\sigma) = \sigma F(\sigma) \qquad (9)$$

erklärt für solche $F(\sigma)$ aus M, für die auch $\sigma F(\sigma)$ zu M gehört.

Satz VIII. *Es lässt sich H_α so isometrisch auf M abbilden, wobei g_α in $F(\sigma) = 1$ übergeht, dass hierbei der Operator A_α in M den Operator K*

induziert. Alle A_α sind also mit K isometrisch äquivalent, also auch untereinander isometrisch äquivalent.

Wir bemerken noch, dass in der Zerlegung (7′) des Elementes h aus H, dem Element h_α aus H_α bei der vorhin besprochenen isometrischen Abbildung eine Funktion $F_\alpha(\sigma)$ aus M entspricht, und somit entspricht jedem Element h eine höchstens abzählbare Menge von Funktionen $F_\alpha(\sigma)$ aus M.

Mit Hilfe dieser Funktionen lässt sich ein expliziter Ausdruck für die Diskrepanz angeben.

Satz IX. *Es seien h und g zwei Elemente aus H, $F_\alpha(\sigma)$ und $G_\alpha(\sigma)$— die ihnen entsprechenden Funktionen aus M, dann gilt:*

$$(D(\delta, \Delta)\, h, g) = -\sum_\alpha \frac{1}{2\pi i\varepsilon} \int_\delta \int_\Delta \frac{F_\alpha(\xi)\,\overline{G_\alpha(\eta)}}{(\xi+\varepsilon i)\,(\eta-\varepsilon i)} \frac{d\xi d\eta}{\xi-\eta} + \frac{1}{2}\sum \int_{\delta\Delta} \frac{F_\alpha(\xi)\,\overline{G_\alpha(\xi)}}{1+\xi^2}\, d\xi; \quad (10)$$

hierbei ist, wenn δ und Δ innere Punkte gemein haben, in den Integralen der ersten Summe der Cauchy'sche Hauptwert zu nehmen.

Auf Grund dieser Ergebnisse, die mit anderen Methoden die Untersuchungen Carleman's (²) verallgemeinern und weiter ausbauen, lässt sich eine allgemeine Theorie der Funktionen eines maximalen Operators entwickeln, deren Darstellung wir einer weiteren Mitteilung vorbehalten.

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MECHANICS

DECAY OF HOMOGENEOUS ISOTROPIC TURBULENCE IN A VISCOUS INCOMPRESSIBLE FLUID

By M. MILLIONSHTCHIKOV

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The method of investigation of turbulence applied in the present paper was proposed in 1924 by Friedman and Keller (1). This method consists in using the differential equations for the coefficients of correlation between pulsations at two different points. Friedman and Keller did not succeed in reaching concrete results by this method. It was only Kármán and Howarth (2), who in 1937/38 obtained by this method the law of the intensity fall of the pulsations with time in the case of a uniform random distribution of the initial turbulent excitations in the infinite space with vanishing mean velocities.

The author considered this case of turbulent movement independently of Kármán and Howarth, and in certain directions succeeded in reaching furthergoing results than these authors. Kármán and Howarth, namely, found the damping law of turbulent excitations in the form of an exponential function with the exponent 5α, where α is an arbitrary constant to be determined experimentally. The author determined the theoretical value of this constant: $5\alpha = -\frac{5}{4}$.

Moreover, below are given explicit analytical expressions for the coefficients of correlation between excitations at two points at any moment. These formulae can be directly verified by experiment. The results of comparison of the formulae of the present note with the experimental data will be published elsewhere *.

In all what follows we assume that the third moments may be neglected (more on the sense of this assumption see further below). This assumption is admissible only for small velocities of the motion, i. e. at the last damping stages of turbulence, when only slow and big eddies remain. At the initial stage of small and fast pulsations the admissibility of this assumption is doubtful. The mentioned result of Kármán and Howarth was also obtained under the same assumption. As to Kármán's and Howarth's attempt in their further investigations to take into account the third moments, neglecting the fourth, it does not appear to be justified. In fact, it could be shown that at a finite damping stage of a homogeneous isotropic turbulence the third moments are

* After this paper was finished I learned that Prof. L. Loitsiansky obtained in his as yet unpublished investigations some of the results of the present note.

small not only in comparison with the second, but also with the fourth ones *. This may be explained as follows. For small velocities all multidimensional distribution laws of the pulsation components in any finite group of points converge to normal laws, and for these the third moments vanish. Hence also in the case of great velocities it is doubtful that the consideration of third moments without the fourth should give good results. Consider now the Navier-Stokes' equations of the motion of an incompressible viscous fluid:

$$\nu\Delta u_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} - \frac{\partial u_i}{\partial t} - \sum_j u_j \frac{\partial u_i}{\partial x_j} = 0, \qquad \sum_j \frac{\partial u_j}{\partial x_j} = 0. \tag{1}$$

Given the initial values for $t=0$ of the functions $u_i(P)$ and $p(P)$ of the point $P=(x_1, x_2, x_3)$, we can determine by these equations the values of these functions for all $t>0$. In studying the turbulence it is natural to consider the initial values themselves as r a n d o m v a r i a b l e s. The general problem of turbulent motion must be in this case formulated as follows: given is the distribution of probabilities for the initial conditions; to determine the distribution of probabilities for the «random functions» $u_i(P)$ and $p(P)$ of the point P at any subsequent moment $t>0$. In the present paper we confine ourselves to considering the second moments of the probability distributions instead of the distributions themselves. The transition from the theory to the experimental data is realized by means of the ergodene hypothesis common to all statistical mechanics: the means over sufficiently long time intervals are near to the corresponding mathematical expectations.

In what follows we shall denote the mathematical expectations by a bar: f. i., $\overline{u_i}$ denotes the mathematical expectation of u_i. By taking the means we shall understand the calculation of mathematical expectations. The deviations of the functions u_1, u_2, u_3, p from their mathematical expectations we shall call p u l s a t i o n s. Put

$$\varphi_i = u_i - \overline{u_i} \ (i=1, 2, 3), \quad \varphi_0 = p - \overline{p}.$$

We introduce also the following denotations for the second moments:

$b_{\alpha\beta} = \overline{\varphi_\alpha \varphi_\beta}$, $B_{\alpha\beta} = \overline{\varphi_\alpha(M')\,\varphi_\beta(M'')}$, and for the third moments:

$$B_{\alpha\beta\gamma}(M', M'', M''') = \overline{\varphi_\alpha(M')\,\varphi_\beta(M'')\,\varphi_\gamma(M''')}.$$

Here M', M'' and M''' denote (generally speaking, different) points of the four-dimensional space:

$$M' = (x_1', x_2', x_3', t'); \quad M'' = (x_1'', x_2'', x_3'', t''); \quad M''' = (x_1''', x_2''', x_3''', t''').$$

With the mode of means-taking adopted by us the necessity of supposing anything about the repeated means disappears, since the repeated taking of mathematical expectations does not give anything new.

Taking the mathematical expectation of the equations (1), we find the equations of Reynolds written for the point M':

$$\left.\begin{aligned} \nu\Delta'\overline{u_i'} - \frac{1}{\rho}\frac{\partial \overline{p'}}{\partial x_i'} - \frac{\partial \overline{u_i'}}{\partial t'} - \sum_j \overline{u_j}\frac{\partial \overline{u_i'}}{\partial x_j'} - \sum_j \frac{\partial b_{ji}}{\partial x_j'} = 0, \\ \sum_j \frac{\partial \overline{u_j'}}{\partial x_j'} = 0 \qquad (i=1, 2, 3). \end{aligned}\right\} \tag{2a}$$

* This remark belongs to A. Kolmogoroff.

This group consists of four equations for ten unknown functions.

In order to close the system of equations under the assumption, on which more will be said below, we must connect the pulsations of the velocity and the pressure at two points of the current.

Using this idea, which belongs to Friedman and Keller, we come to equations containing the moments of the third order. Supposing that we may neglect the third moments $B_{\alpha\beta\nu}$, we obtain the following system of equations of turbulent motion of an incompressible viscous fluid:

$$\left.\begin{aligned} \nu\Delta' B_{ik}(M', M'') - \frac{1}{\rho}\frac{\partial B_{0k}(M', M'')}{\partial t'} - \sum_j \overline{u_j'}\frac{\partial B_{ik}(M', M'')}{\partial x_j'} - \\ - \sum_j \frac{\partial \overline{u_i'}}{\partial x_j'} B_{jk}(M', M'') = 0, \\ \sum_j \frac{\partial B_{jk}(M', M'')}{\partial x_j'} = 0, \qquad \begin{matrix}(i=1,2,3)\\ (k=0,1,2,3)\end{matrix} \end{aligned}\right\} \quad (2b)$$

which is written for the point M'.

The system (2a — 2b) contains twenty differential equations for twenty unknown functions. A similar system of equations may also be written for the point M''. [In this form the equations (2b) were given by A. Kolmogoroff in his communication at the Conference of the Institute of Mechanics of the Moscow State University in December 1936].

We shall write both systems only for the case in which we are interested, when $\overline{u_i} = 0$ $(i = 1, 2, 3)$ in the whole space.

$$\left.\begin{aligned} \nu\Delta' B_{ik}(M', M'') - \frac{1}{\rho}\frac{\partial B_{0k}(M', M'')}{\partial x_i'} - \frac{\partial B_{ik}(M', M'')}{\partial t'} = 0, \\ \sum_j \frac{\partial B_{jk}(M', M'')}{\partial x_j'} = 0, \\ \nu\Delta'' B_{ik}(M'', M') - \frac{1}{\rho}\frac{\partial B_{0k}(M'', M')}{\partial x_i''} - \frac{\partial B_{ik}(M'', M')}{\partial t''} = 0, \\ \sum_j \frac{\partial B_{jk}(M'', M')}{\partial x_j''} = 0. \end{aligned}\right\} \quad (3)$$

We shall try to find the solution of the system (3) for the symmetrical process of homogeneous isotropic turbulence, for which the functions B_{ik} depend only on the differences of the space co-ordinates of the points M', M'' (homogeneity) and are invariant with respect to the rotations of the system of co-ordinates about its axes and to the reflexions of the system of co-ordinates in planes passing through its origin (isotropy) ([2],[3]).

From the definition of isotropy it follows that for two points M' and M'' lying on the axis OX_1 only four correlation functions can be different from zero, namely: $B_{11} = B_{dd}$; $B_{22} = B_{33} = B_{nn}$; $B_{01} = B_{0d}$; B_{00}.

It may be proved that in the general case of points lying on an arbitrary straight line with directing angles α_1, α_2 and α_3 there exists the following connexion between the different correlation functions:

$$B_{ik} = \overline{B}\cos\alpha_i\cos\alpha_k + \delta_{ik}B_{nn}, \quad B_{0k} = B_{0d}\cos\alpha_k, \quad (4)$$

where $\overline{B} = B_{dd} - B_{nn}$, $\delta_{ik} = 1$ for $i = k$, $\delta_{ik} = 0$ for $i \neq k$.

From the equations (3) and the relations (4) we conclude that $B_{0k}=0$ for $k=0, 1, 2, 3$ and the system (3) may be reduced to the two equations

$$\nu\left(\frac{\partial^2 B_{dd}}{\partial r^2}+\frac{4}{r}\frac{\partial B_{dd}}{\partial r}\right)=\frac{\partial B_{dd}}{\partial \tau} \quad * \tag{5}$$

where $\tau=t'+t''$ and

$$\frac{\partial B_{dd}}{\partial r}=-\frac{2}{r}\overline{B}. \tag{6}$$

The solution of equation (5) can be easily found, since the operator on its left hand side is the operator of Laplace in the five-dimensional space applied to the function B_{dd} depending only on r and τ. If we put $r=\sqrt{y_1^2+y_2^2+y_3^2+y_4^2+y_5^2}$, we easily become convinced that the equation (5) is equivalent to the following equation:

$$\nu\sum_j \frac{\partial^2 L_{dd}(y_1, y_2, \ldots, y_5, \tau)}{\partial y_j^2}=\frac{\partial L_{dd}(y_1, y_2, \ldots, y_5, \tau)}{\partial \tau}, \tag{7}$$

where $L_{dd}(y_1, y_2, \ldots, y_5, \tau)=B_{dd}(r, \tau)$.

The solution of the equation (7) assuming for $\tau=0$ the initial value $L_{dd}^{(0)}$ is known and has the form

$$L_{dd}(y_1, y_2, \ldots, y_5, \tau)=$$

$$=\frac{1}{(2\nu\pi\tau)^{\frac{5}{2}}}\int_{-\infty}^{+\infty}\cdots\int L_{dd}^{(0)}(z_1, z_2, \ldots, z_5)\, e^{-\frac{\sum (y_j-z_j)^2}{4\nu\tau}}\, dz_1 \ldots dz_5, \tag{8}$$

or, since $L_{dd}^{(0)}=B_{dd}^{(0)}(\rho)$ and $\tau=t'+t''$,

$$B_{dd}(r, t', t'')=$$

$$=\frac{\exp\left(-\frac{r^2}{4\nu(t'+t'')}\right)}{r^{\frac{3}{2}}\, 2\nu(t'+t'')}\int_0^{\infty} B_{dd}^{(0)}(\rho)\,\rho^{\frac{5}{2}}\, I_{\frac{3}{2}}\left(\frac{r\rho}{2\nu(t'+t'')}\right)\exp\left(-\frac{\rho^2}{4\nu(t'+t'')}\right)d\rho. \tag{9}$$

Of special interest is the case when at the initial moment only very fine eddies exist, i. e. when the functions $B_{dd}^{(0)}(\rho)$ and $B_{nn}^{(0)}(\rho)$ are noticeably different from zero only for very small ρ. In this case we may apply the limiting solution corresponding to a «point source». This solution is expressed by the elementary formula:

$$B_{dd}(r, t', t'')=\frac{k}{[2\nu(t'+t'')]^{\frac{5}{2}}}\exp\left(-\frac{r^2}{4\nu(t'+t'')}\right). \tag{10}$$

From (10) and (6) follows

$$B_{nn}(r, t', t'')=\frac{k}{[2\nu(t'+t'')]^{\frac{5}{2}}}\left[1-\frac{r^2}{4\nu(t'+t'')}\right]\exp\left(-\frac{r^2}{4\nu(t'+t'')}\right). \tag{11}$$

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* Cf. Kármán [2].

The formulae (10) and (11) solve the following physical problem: at the initial moment the infinite space is filled with very fine eddies situated at random which arose at different points of the space independently of each other (we mean the independence in the usual sense of the theory of probabilities); moreover, they serve to determine under these assumptions the mean squares of the intensity pulsations and the coefficients of correlation between the pulsation components at different points and at different subsequent moments.

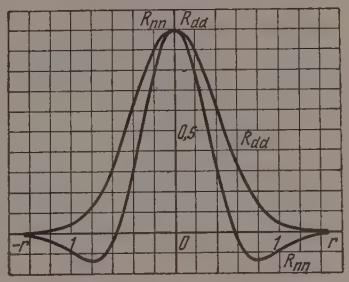

Fig. 1. R_{dd} and R_{nn} as functions of r for $8\nu t = 0.33$.

For the mean value b_{ii} of the square of the velocity pulsation component v_i at the moment t we find

$$b_{ii}(t) = b(t) = B_{dd}(0, t, t),$$

or, in virtue of the formula (11),

$$b_{ii}(t) = b(t) = \frac{k_1}{t^{\frac{5}{2}}}, \qquad (12)$$

where $k_1 = \frac{k}{(4\nu)^{\frac{5}{2}}}$ is a constant depending on the initial intensity of the pulsations. The correlation coefficients $R_{ij}(M', M'')$ between $v_i(M')$ and $v_j(M'')$ may be expressed in a way similar to (4) through the correlation coefficients

$$R_{dd}(r, t', t'') = \frac{B_{dd}(r, t', t'')}{\sqrt{b(t')\,b(t'')}}; \quad R_{nn}(r, t', t'') = \frac{B_{nn}(r, t', t'')}{\sqrt{b(t')\,b(t'')}}$$

between the longitudinal and transversal velocity components at two points situated at the distance r. In particular, for $t' = t'' = t$ we have

$$R_{dd}(r, t, t) = \exp\left(-\frac{r^2}{8\nu t}\right); \quad R_{nn}(r, t, t) = \left(1 - \frac{r^2}{8\nu t}\right)\exp\left(-\frac{r^2}{8\nu t}\right). \qquad (13)$$

It is interesting that while R_{dd} is positive for all r, the correlation coefficient R_{nn} is positive only for $r^2 < 8\nu t$ (Fig. 1). Experimentally this circumstance was discovered by Dryden ([4]).

In conclusion it is my duty to express my gratitude to Prof. A. Kolmogoroff for directing my attention to this problem and for his valuable advices during the preparation of the present paper.

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MECHANICS

DECAY OF TURBULENCE IN WIND TUNNELS

By M. MILLIONSHTCHIKOV

(Communicated by S. N. Bernstein, Member of the Academy, 28. XII. 1938)

The honeycombs of wind tunnels which serve successfully their fundamental purpose — smoothing the field of velocities — have also a negative effect: they are complementary turbulizors of the air stream.

On the so-called «initial turbulence»

$$w'_0 = \frac{\sqrt{\overline{v_0^2}}}{u}$$

the turbulence caused by the honeycomb is superposed. In the first approximation we may assume that this superposition is expressed by the simple summation:

$$w' = w'_0 + w'_{\text{hon}}.$$

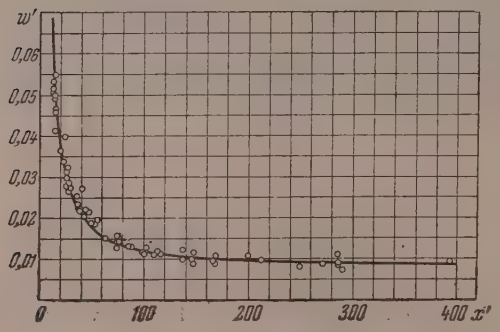

Fig. 1. Decay of turbulence behind the honeycomb. Theoretical curve $w' = w'_0 + cx'^{-\frac{5}{4}}$ ($w_0 = 0.008$, $c = 1.3$) and the experimental points of Dryden.

It is natural that w'_{hon} should be damped in the course of time, since we have before us a decay of turbulence under the influence of viscosity.

The conditions in the central part of the air stream

$$\overline{u}_1 = \text{const}, \quad \overline{u}_2 = \overline{u}_3 = 0$$

enable us to apply to the problem of decay of the pulsations behind the honeycomb the theory of isotropic turbulence (1).

For the mean value $b(t)$ of the square of the velocity pulsation component v_i at the moment t we have then [formula (12), (1)]:

$$b(t) = \frac{k_1}{t^{\frac{5}{2}}}.$$

Instead of the time t we may introduce the distance from the honeycomb, according to the formula $t = \frac{x}{u}$. As usual, we introduce into consideration the relative distance from the honeycomb $x' = \frac{x}{M}$, where M is the size of the honeycomb cells, and the relative pulsation $w'_{\text{hon}} = \frac{\sqrt{b(t)}}{u}$, and obtain

$$w'_{\text{hon}} = cx'^{-\frac{5}{4}}$$

where c is a new arbitrary constant. Then the expression for the total pulsation will have the form

$$w' = w'_0 + cx'^{-\frac{5}{4}}.$$

Comparison with Dryden's experiment ([2]) shows that for $c = 1.3$ ($w'_0 = 0.008$) the experimental points fit the curve (Fig. 1) very well, i. e. the obtai-

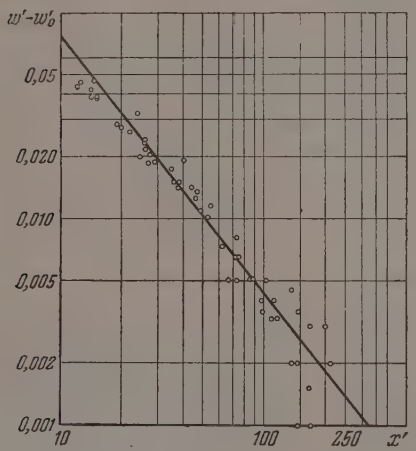

Fig. 2. Curve and points of Fig. 1 in logarithmic scale.

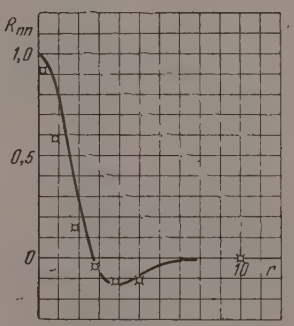

Fig. 3. R_{nn} as function of r for $8\nu t = 7.56$ and experimental points of Dryden.

ned exponent $-\frac{5}{4}$ does in fact characterize the decay of turbulence in the air stream. Particularly well it can be seen on logarithmic paper (Fig. 2).

As to the comparison with experiments of the variation of the correlation coefficient $R_{nn}(r, t)$ with the distance r, we may naturally expect here coincidence in the domain of fine vorticity.

Dryden ([2]), who carried out measurements in the domain of fine vorticity, found the character of variation of the correlation coefficient R_{nn} to correspond to our theoretical result (Fig. 3). This figure shows the curve R_{nn} in dependence of r for $8\nu t = 7.56$ and the experimental points found by Dryden.

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CHEMISTRY

ON THE QUESTION OF THE OVERVOLTAGE OF SILVER

By A. T. VAHRAMIAN

(Communicated by A. N. Frumkin, Member of the Academy, 21.XII.1938)

In the literature there is a number of investigations dealing with the dependence of the polarization of metals upon the current density. However, in all these investigations the magnitude of the polarization is considered as a function of the apparent current density d_1, which is determined by the ratio of the current strength to the geometric surface of the cathode.

Using the electrolytic deposition of silver as an example, we showed(1) experimentally that the current density d_1 does not coincide with the current density d_2, which is determined by the ratio of the current strength to the active part of the cathode surface*. No relation could be found between the values d_1 and d_2.

The value of the polarization is a function of the current density d_2 only; and it is therefore only natural that polarization data for one and the same metal, which are presented in the literature, do not coincide (Fig. 1).

Curve *1* in Fig. 1 represents the results of measurements by Erdey-Gruz and Volmer(2) for a one-normal solution of $AgNO_3$; Curves *2* and *4* were obtained by Heckstra(3); while Curve *2* represents results of experiments in which the surface of the cathode was continually being scraped off during the electrolysis, Curve *4* was obtained from experiments with the usual electrode. We obtained Curve *3* by plotting the data of Samartzev and Evstropyev(4). Curves *5* and *6* were obtained by Erdey-Gruz and Volmer(5) for the solution 0.05 *n* $AgC_2H_3O_2$ and 0.1 *n* $HC_2H_3O_2$, etc.

However, the experimental investigation of the dependence between the polarization value and the current density d_2 is made difficult owing to the presence of the following regularity. At the moment when the current strength begins to change, the active cathode surface starts changing also, and current density d_2 gradually assumes its former value. Thus, by counting and measuring the crystals during the electrolytic precipitation of silver, Aten and Boerlage (6) showed that the actual current density remains constant when the current strength in the circuit increases, and the apparent current density correspondingly increases also. Later Kohlschütter and Torricelli (7) showed that if the potential on the electrodes is kept constant, the current strength (i) adapts itself to the active surface (S) of the electrode $[i=f(S)]$. And finally, Samartzev(8) observed that during the transition from one constant current strength to another, the active surface adapts itself to the current strength $[S=F(i)]$.

* The active surface is arbitrarily taken as that part of the surface on to which the metal precipitates.

Using Ag and Pb as an example, we showed ([9]) that this relation also exists during the polycrystalline precipitation of metals. The presence of this relation prevents the normal change in the current strength and the establishment of the desired value of the current density d_2.

To circumvent the difficulty we made use of the fact that the increase in the active cathode surface and restoration of the former value of the current density takes place not at once after the change in the current strength, but during a certain time interval. It is known that at the first moment after an increase in the current strength, its density correspondingly increases, since the active cathode surface had not had the necessary time to change. To establish the relation between the current density d_2 and the value of the polarization, we used those points, from the curves of the change in polarization with time for various current densities, which corresponded to the first moment after the change in the current strength (Fig. 3, points $б'$, $г'$, and $е'$).

The procedure used in the present study was the same as that described in our previous paper ([9]).

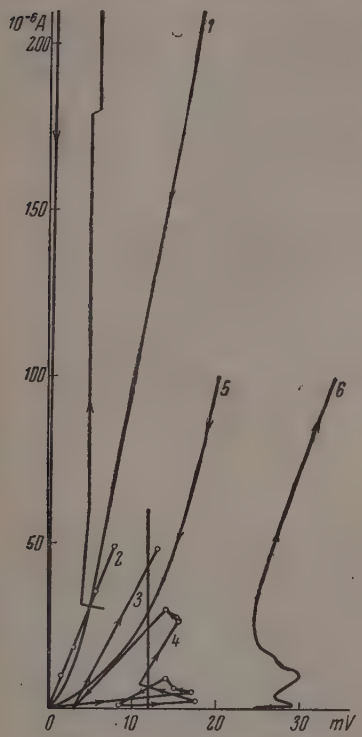

Fig. 1.

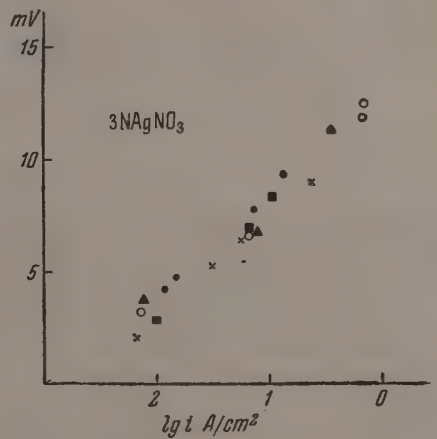

Fig. 2.

The measurements of the overvoltage were carried out in a 3 n solution of $AgNO_3$ within the current density d_2 range from 0.0075 to 0.7 A/cm². The accuracy in measuring the overvoltage is chiefly dependent upon the accuracy with which the active cathode surface can be measured. The results obtained are presented in Fig. 2*.

As is seen from Fig. 2, the values for the polarization which we obtained markedly differ from the values obtained by the authors cited above; our values in their absolute magnitude are well below those obtained by these authors. The dependence of the value of the polarization upon the logarithm of the current density was found to be linear.

* For these experiments the electrolyte was not purified very carefully since in completely pure solutions the thread-like growth of a crystal is destroyed, and therefore a determination of the active cathode surface becomes very inaccurate. In this case the incline of the curve depends on the purity of the electrolyte and presence of oxygen.

As has been stated above, the data for the overvoltage on silver, obtained by different authors, do not agree with each other. Not only the absolute values of the polarization are different, but also the law according to which the polarization changes with the current density.

To find out the reasons for such a difference in the results, it is necessary to examine how the active cathode surface, and hence the current density d_2, changes during electrolysis. It is known that during the electrolytic precipitation of silver from $AgNO_3$, using a strictly constant current strength, the crystal growing on the cathode assumes the shape of a thread with a constant cross-section. This part of the crystal is diagrammatically represented in Fig. 3 by the sector *аб*. The current density d_2 is determined by the ratio of the current strength to the cross-sectional area of the thread. As the current strength in the circuit increases, the thread gradually becomes thicker. The value of this part of the crystal, which is designated in Fig. 3 by *бв*, depends upon the composition of the electrolyte and upon impurities present in it. On the average, the thread grows in thickness for about 30 minutes.

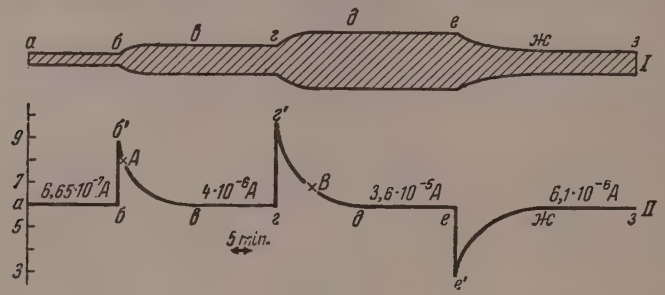

Fig.3.

After the initial value of the current density d_2 is established, the thickening of the thread ceases, and it continues to grow further with a greater, constant cross-section (in Fig. 3 this corresponds to *вг*). Any further increase in the current strength has similar consequences.

The current density in sectors *аб*, *вг*, *де* and *жз* is the same. Fig. 3, *I*, which diagrammatically pictures the shape of the growing crystal, is given for the purpose of comparison with the curve showing the dependence of the change in polarization with time upon a change in the current strength (see Curve *II*). Both on the curve and in the schematic drawing of the crystal, the sectors corresponding to the same interval are denoted by the same letters. The apparent current density in sectors *аб*, *вг*, *де* and *жз* is different, since, with the initial cathode surface constant, the current strength changed several times during the electrolysis. The current density d_2 is, however, constant for all these sectors. From this point of view it is comprehensible that the value of the polarization corresponding to these sectors (sectors *аб*, *вг*, *де* and *жз*, Curve *II*) should be the same. On the other hand, such a constancy of the polarization value can by no means be explained on the basis of the apparent current density d_1. The apparent current density at the point *A* (Curve *II*, Fig. 3) is considerably smaller than at the point *B*, while the value of the polarization at the point *A* is higher than at the point *B*. If however the actual, and not the apparent, current density d_2 be considered, there will be nothing particular about the ratio of polarization values at these points, for the current density d_2 at the point *A* is higher than at the point *B*.

From the schematic drawing of the growing crystal *I* and Curve *II* of Fig. 3 it is seen that the results obtained are highly variable, depending on

the rate at which the current strength in the circuit was changing and on the interval elapsed between the time when it started to change and the time when polarization was measured. If polarization is measured right after the increase of the current strength, a larger value will be obtained; while a measurement of the polarization, carried out when a certain time interval has passed after the change in the current strength, but before the self-adapting current density is established, will give a smaller value of the polarization. Finally, if polarization is measured after the self-adapting current density is established (after points *е*, *∂* and *эк*), the initial constant value will be obtained.

All the authors mentioned above attempted to establish the dependence of the value of the polarization upon the apparent current density. We have shown that there is no direct relation between these quantities; and, depending upon the interval of time which passes between the change in the current strength and the measurement of the polarization, the apparent relation between them will be expressed in a totally different fashion.

The actual relation between the current density and the value of the polarization cannot possibly be established unless current density is expressed by the ratio of the current strength not to the total surface of the cathode, but only to the active surface of the growing crystal.

The investigation of the dependence of the value of the polarization upon the actual current density d_2 under different conditions is being continued.

I should like to take this opportunity to express my gratitude to Prof. A. Frumkin for his advice, which has made possible to accomplish this work.

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CHEMISTRY

SYNTHESIS OF INDERITE

By I. B. FEIGELSON, V. E. GRUSHVITSKY and T. V. KOROBOCHKINA

(Communicated by N. S. Kurnakov, Member of the Academy, 24.XII.1938)

It was in course of studying the genesis of minerals from Inder deposit that an investigation of reciprocal solubility of magnesium sulphate and borax at +35 and +50°C, i. e. in certain points of the system $2NaCl+MgSO_4 \rightleftarrows Na_2SO_4+MgCl_2$, complicated by the introduction of sodium tetraborate, was started by I. B. Feigelson in the Research Station for Salts at Saratov. The solubility of borax was found thereby to go down as the concentration of magnesium sulphate increased.

In the precipitates formed after equilibrium had established itself in the solution, boric anhydride and magnesium were estimated. Examination under the microscope (M. N. Godlevsky) showed the crystals to have such refraction indices as are characteristic of the natural inderite described by A. M. Boldyreva and E. N. Egorova (1).

Farther and more detailed work was necessary by whose means the proper conditions under which the formation of the salt detected takes place, its chemical composition, optical constants and other physico-chemical properties might be determined. This was done at the Institute for Salt Research, the experimental part of the work being performed by T. V. Korobochkina.

There was prepared a solution of 210.3 g $Na_2B_4O_7 \cdot 10H_2O$; 157.5 g $MgSO_4 \cdot 7H_2O$; and 2632 g water, the total being 3000 g.

The container which held the solution was a jar of 3.5 l capacity, stopped and provided with an oil lock and a glass stirrer. It was kept in a water thermostat at +35°C. At first, the two salts dissolved completely and the liquid was clear for three days. After this time elapsed, a crystalline precipitate was observed to form on the walls of the jar and on the stirrer, which stuck fast to the glass surface.

Samples from this precipitate were taken repeatedly 25 and 28 days later. Examined under the microscope they invariably showed a mixture of two kinds of crystals. One of them, whose amount was well below that of the other, proved to have the same refraction indices, as formerly established by M. N. Godlevsky (2).

When the liquid phase, drawn out by a syphon, was poured in the jar again, we saw the solution to grow turbid throughout and to deposit fine crystals. We allowed it to stand for 24 days more and then took samples both of the liquid phase and abundantly precipitated crystals, which were now easy to release from the glass.

Part of the moist precipitate was being washed with cold water until no reaction in it for ion SO_4 was observable. Thereupon it was washed with alcohol and dried in air. As proved by control examination under the microscope, washing failed to induce any change whatever in the properties of the crystals which before and after it showed the same indices, coinciding with those of the natural inderite, and no other salt could be detected in them.

The samples consisted of long columnar or fine fibrous orthorhombic crystals, optically biaxial with the angle 2 V near to 60°. The indices of refraction were: N_g=1.505($\pm$0.002) and N_m =1.489 ($\pm$0.002), while those of the natural inderit are: N_g=1.504 and N_m=1.488.

The crystals gave nearly straight extinction and were optically positive.

A quite regular conjunction was observed parallel to the macro-axis; they were polysynthetic twins with the twin-plane parallel to the above axis (crystallographic examination was performed by A. S. Zaporozhtzeva and P. V. Grushvitsky). The specific weight of the salt, estimated in water, was also in good agreement with the data of Boldyreva and Egorova. It was 1.78 against 1.79, which is the figure established by the above-mentioned authors.

The precipitate washed with water and alcohol was subjected to chemical analysis. Magnesium was estimated by weight according to Schmitz; boric anhydride, by means of usual titration with decinormal alkali, phenolphtalein and inverted sugar.

Water was estimated by calcinating the test-portion at 500°C, until its weight became constant and then the content of boric anhydride was checked in order to be sure that no volatilization of it took place under the conditions available. The results are referred to in the table along with the data for natural inderite and the theoretical composition.

Composition by weight in %	B_2O_3	MgO	H_2O	Admixtures	Total
Synthetic inderite	37.31	14.36	48.88	—	100.55
Natural inderite	35.60	14.34	48.20	2.62	100.76
Theoretical composition	37.32	14.40	48.28	—	100.00

52 days after the beginning of the experiment, when the samples from the precipitate were taken, the composition of the liquid phase was as follows: MgO, 0.35%; B_2O_3, 1.21%; SO_3, 1.85%; Na_2O, 1.23%.

From the figures of chemical composition referred to above we are entitled to say that the salt obtained is a synthetic inderite corresponding to the formula $2MgO.3B_2O_3.15H_2O$, while the optical data bear out its identity with the natural mineral described by A. M. Boldyreva and E. N. Egorova.

The present work is a new contribution to the series of investigations carried out at the Institute of General and Inorganic Chemistry of the Academy of Sciences of the USSR by A. V. Nikolaev on the synthetic preparation of other minerals from the Inder deposit([3],[4]). The work is being continued with the purpose of further inquiry into the physico-chemical properties of the artificial inderite as well as to establish the range of concentrations and temperatures at which its formation takes place.

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CHEMISTRY

OCCURRENCE OF BORON IN SALT LAKES OF ARAL-KASPIAN REGION

By I. B. FEIGELSON, M. G. VALIASHKO and A. G. BERGMAN

(Communicated by N. S. Kurnakov, Member of the Academy, 27.X.1938)

The geochemical investigations of boron by V. M. Goldschmidt and Peters ([1]) have established its widespread occurrence in sediment rocks. This gave rise to a number of works on the distribution of boron, their authors being prompted not only by the theoretical interest of the problem, but also by the steadily growing demand for boron compounds on the side of the industry and, since lately, of the agriculture too.

Distribution of boron in brines of S. O. Lakes of the USSR (according to investigations of the Saratov station far Salt Research).

In this country the research work on boron received special impetus from the fact that in 1934 workable boron deposits were discovered in the Inder mountains. Among the works referring to this period particular interest is due to those in which the occurrence of boron in waters and brines of the salt lakes, so numerous in the USSR, is being considered as one of the phases of its migration in the earth crust (hydrochemistry of boron).

The estimation of boron content in superficial waters has, however, also a practical meaning since this may sometimes prove a valuable prospect for the presence of boron deposits.

In the summer of 1938 the Saratov salt station, while pursuing its physico-chemical investigation of the salt lakes in Western Kazakhstan and

adjoining Republics ([2–6]), was successful in carrying out a large-scale reconnaissance which embraced the lakes between the Aral and Caspian, in the left-sided Khorzem and farther up the river Amu-Daria to Chardjou.

The present communication is the summary of the results obtained from the estimation of boron in the samples supplied by this expedition.

Boron was estimated according to V. N. Tageieva ([7]), in no less than two parallel test-portions (50—1000 ml brine each).

The results of the analysis, recalculated to the content of B_4O_7 in 100 ml brine, are summarized in the table, column 6.

In order to compare the content of boron in waters of different concentrations we expressed it in grams per 1000 g of salt complex (mineral residue), this being done by means of the following formula:

$$K_B = \frac{[B_4O_7] \text{ g/100 ml}}{[\sum \text{of salts}]\,\% \times d} \times 10^3.$$

The values of K_B are given in column 7.

On ground of the data referred to in the table all the lakes prospected can be divided into the following groups according to their K_B values.

1) K_B reduced 0.02—0.25 . . . . . Lake of Plateau Ust-Yurt; lake Tuz-Kyr in Khorzem

2) K_B middle 0.25—0.6 . . . . . Lakes of Chelkar group; lakes of Kok-Chag in Khorzem; lakes 1 and 2 of Kara-Khodja (Deinausk region)

3) K_B increased 0.6—1.0 . . . . . Lakes of Sara-Kamysh depression

4) K_B high > 1 . . . . . Lakes of Khivinsky region; lake Khazarasp; lake Sultan-Sandjar with feeding sources; lake Djaman-Sor in Chelkar region; lake Karagach-Mayak in Deinausk region (Chardjou)

For none of those lakes, except that of Sultan-Sadjar([8]), are data on the content of boron available in literature.

We can thus mark out a region of lakes south of Khorzem, including the lakes of Sultan-Sandjar, in which high content of boron has been revealed. If we were to take the values K_B as a measure to compare this region with those where the deposits of boron minerals are beyond doubt, we will find it to equal if not to exceed even the Inder deposit itself with its K_B=2.10—1.36.

The lakes lie on the very boundary between the Oasis and Kara-Kum sands, in South Khorzem. The region is worth detailed investigations in geochemical and geological aspects.

It will not be amiss to point out here that the increased content of boron seems to be a particular of the salt lakes both in the present and ancient basin of the river Amu-Daria (the lakes near Chardjou in Deinausk region, the group of lakes of Sultan-Sandjar, the lake of Sara-Kamysh depression). Among other lakes with high K_B values two are most prominent, viz., Djaman-Sor lake of Chelkar group in the south and Karagach-Mayak lake of Deinausk region in the north.

On the other hand, the lakes of Plateau Ust-Yurt and the lake Tuz-Kyr in Khorzem are characterized by very low values of K_B.

The material collected supplies new data on the occurrence of boron in the brines of salt lakes and sets before us the task to inquire into the geochemical processes which lead to such a distribution of boron.

Boron Content in Brines of Salt Lakes in Kazakh, Turkmen and Uzbek Soviet Socialist Republics

(data supplied by the expedition of Saratov Research Station for Salts in 1938)

Sampling No	Lake and place of sampling	Region	Date of sampling, 1938	Salt sum in % by weight	B_4O_7 per 100 ml brine	K_B
1	The river Uil, near Temir bridge	Ural-Emba	1.VII	0.1	0.0005	5.0
2	Mulaly-Tuz lake, in the central part of the lake	Chelkar	6.VII	29.2	0.0372	0.20
3	Sor-Sulfatnik, near the 4th hamlet	»	6.VII	17.51	0.0127	0.60
4	Balgasin lake	»	6.VII	27.33	0.0116	5.35
6	Unknown lake, 20 km from St. Chelkar	»	6.VII	28.73	0.0074	0.21
7	Djaman-Sor lake	»	6.VII	29.78	0.0518	1.39
8	Chelkar lake	near town Chelkar	7.VII	0.142	0.0008	5.28
10	Kichkino-Tuz lake	Chelkar	8.VII	26.59	0.0165	0.50
11	Kurgan-Tuz lake	»	8.VII	27.21	0.0144	0.43
12	Kurgan-Tuz lake off the north shore in the river assembling spring waters	»	8.VII	22.23	0.009	0.34
13	Chulak-Dzide lake, central part of the south shore	»	8.VII	27.07	0.0147	0.45
14	Kara-Batyr lake, east part	»	9.VII	17.93	0.0035	0.18
15	Kashkar-Ata lake	»	10.VII	26.63	0.0105	0.33
18	First Sor lake, near lower Ust-Yurt	Plateau Ust-Yurt	13.VII	26.57	0.0058	0.18
20	Sam lake, westerly part	Do	14.VII	26.96	0.0036	0.11
21	Asmanty-Matay lake	»	15.VII	26.34	0.0028	0.09
23	Asmanty-Matay lake, spring issue in the north shore	»	15.VII	8.39	0.0043	0.07
24	Kos-Bulak lake, N. E. part	»	16.VII	26.17	0.0006	0.02
25	Kos-Bulak lake, N. E. part	»	16.VII	28.32	0.002	0.06
26	Aral Sea, west shore, middle part	»	17.VII	1.07	0.0011	1.10
27	Sara-Kamysh lake III	Sara-Kamysh depression	26.VII	29.23	0.0359	0.99
30	» » » I	Do	26.VII	28.06	0.0191	0.57
31	Tuz-Kyr lake, south part	Leninsk (Ak-Tyupe, TSSR)	1.VIII	27.49	0.0059	0.20
32	Tuz-Kyr lake, south part from the bore-hole	Do	1.VIII	27.93	0.0071	0.20
35	See-Kul lake	Ilialinsk (TSSR)	2.VIII	5.17	0.0016	0.29
38	Kok-Chaga lake	Do	2.VIII	27.97	0.0104	0.31
40	Kara-Kul lake, east bank	Chiva (Uzbek SSR)	3.VIII	0.163	0.004	5.98
41	Shor-Kul lake	Do	4.VIII	2.40	0.0041	1.68
42	Khazarasp lake	town Khazarasp	5.VIII	20.05	0.0344	1.40

Sampling No	Lake and place of sampling	Region	Date of sampling, 1938	Salt sum in % by weight	B_4O_7 per 100 ml brine	K_B
43	Shikh-Aryk	Pitnyak Uzbek SSR	5.VIII	11.67	0.0083	0.60
44	Sultan - Sandjar lake, spring № 1, near resort .	Dargana-tinsk TSSR	6.VIII	0.3*	0.0045	1.52
45	Sultan - Sandjar lake spring № 2	Do	7.VIII	0.6*	0.0054	9.02
46	Sultan - Sandjar lake, spring Khyrsbulak, north-west shore . . .	»	7.VIII	0.8*	0.0066	8.21
47	Sultan - Sandjar lake, from a salt brine pool, in front of the spring № 7	»	7.VIII	27.3*	0.1780	5.29
48	Sultan-Sandjar lake, peninsula Kurort, spring № 8	»	7.VIII	0.5*	0.0041	8.10
49	Sultan - Sandjar lake, Agar-Bulak spring . .	»	7.VIII	0.6*	0.0040	6.63
50	Sultan-Sandjar lake, Sassyk-Bulak spring . .	»	7.VIII	0.6*	0.0044	7.31
51	Sultan - Sandjar lake, from the spring on the south - west slope of Kok-Bulak	»	7.VIII	0.5*	0.0044	8.80
52	Sultan Sandjar lake, north bore hole . . .	»	8.VIII	28.64	0.1313	3.80
53	Sultan Sandjar lake, Kaisha-Bulak spring, near «3 kibitki»	»	8.VIII	0.7*	0.0045	6.46
54	Sultan Sandjar lake, Kamysh - Bulak spring, N. W. from Kok-Bulak	»	8.VIII	0.6*	0.0046	7.60
56	Kaporas lake	»	9.VIII	28.00	0.0167	0.51
58	» »	»	9.VIII	28.17	0.0164	0.50
61	Amu-Daria near Chardjou, hydrolog. stat. № 4753	Chardjou	15.VIII	0.03—0.06	0.00022**	7.33—3.66
62	Kara-Khodja lake № 1, middle part	Deinausk (TSSR)	18.VIII	28.65	0.0074	0.21
63	Kara-Khodja lake № 2, middle part	Do	18.VIII	29.52	0.0122	0.33
64	Karagach-Mayak, middle part	»	18.VIII	31.19	0.0419	1.00

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* Owing to the very small content of boron the analysis is approximate.

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GEOCHEMISTRY

CONTRIBUTION TO THE QUESTION OF THE METAMORPHISM OF THE SUBTERRANEAN WATERS OF LEVSHINO*

By A. M. KUZNETZOV

(Communicated by W. I. Vernadsky, Member of the Academy, 24.XII.1938)

In studying the physico-chemical properties of the subterranean waters of Levshino, we came across the following highly interesting fact: at the same level there were found to exist waters differing in composition and mineralization, as well as waters of the same type but in rocks of lithologically different description. These waters deserve particular attention (1) also on account of the fact that at a depth of but 40—50 m there are to be found calcium chloride and magnesium brines with a high content of potassium, bromine and iodine. The fact that these waters are well identified and that a large volume of representative material on them is available affords rare possibility for solving the question of their genesis.

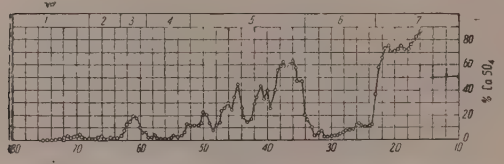

Fig. 1. Distribution of $CaSO_4$ in the thickness of rocks: *1*—sandstone-argillaceous, *2*—flag limestones, *3*—light dolomites, *4*—calciferous marls, *5*—gypsiferous marls and dolomites, *6*—argillaceous and striated dolomites and *7*—gypsums and anhydrites. The curve is plotted according to 1000 analyses for the determination of SO_4 in various rocks with a complete sampling of the core of 10 wells.

The subterranean waters of Levshino are running in the Kazan and Upper Kungur deposits (under the flood plain and bed of the river Kama) at a depth of 80 m. The rocks have a slight southward dip with a complete absence of disjunctive tectonic disturbances. The strata of the Upper Kungur rocks in the southern part of the region are bedded under a thick layer of Kazan deposits, while in the northern part, where the latter are washed off by erosion, the heads of these deposits are hidden under alluvial sediments.

The Kungur rocks (marls and dolomites) are characterized by a considerable content of gypsum and a small content of sodium chloride, the latter being, however, available also in gypsums and anhydrites. The property of emitting the smell of hydrogen sulphide after being fractured is a peculiar feature of the anhydrite, gypsum dolomites and marls of the lower zones.

The distribution of gypsum in the thickness of the layer testifies to a

* Within 20 km from the town of Perm up the river Kama.

rhythmical damping down of the deposits of calcium sulphate, as recorded in the chronicles of sedimentation in the Kungur Sea (Fig. 1).

All the rocks within the surveyed field (20 km²) are fissured and water-bearing. The fissuriness, having a predominant meridional direction, reaches its utmost development in the valley of the river Kama, at the place where the water-tight roof of the bed rocks (argillites) has been washed off by erosion. Per contra, fissuriness, as we proceed from the outcrop, damps down in the direction of the basic boundaries and the dip. Thus, the subterranean waters are confined to a definite locality—they belong to the valley of the Kama and are functionally connected with fissuriness.

The existence of fissural waters should bring evidence to bear in favour of their being most variously mineralized, since the latter factor is dependent upon the length of the path travelled by running water along a fissure, as well as upon the moment of its beginning to percolate into the rock. As a matter of fact, a great variation in the content of dissolved substances was observed, ranging from fractions of 1 g to 150 g per litre. However, mineralization tends to grow in a regular manner according to the depth of the waters and is accompanied by sharply marked changes in chemical composition.

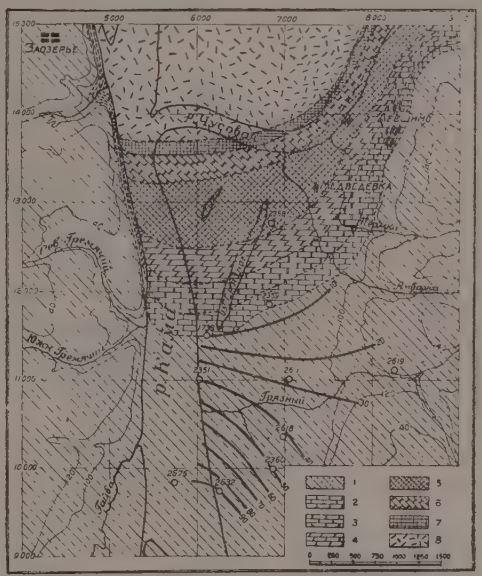

Fig. 2. Geological map of the district of Levshino with isolines of mineralization of the waters of argillaceous and striated dolomites: *1*—sandstone-argillaceous strata, *2*—flag limestones, *3*—light dolomites, *4*—calciferous marls, *5* gypsiferous marls and dolomites, *6*—argillaceous dolomites, *7*—striated dolomites and *8*—gypsums and anhydrites.

The chemical composition of the waters is highly variable and is so intimately bound up with mineralization, that there is no chance of detecting the presence of sodium sulphate in the brines, or of magnesium and calcium chloride in the waters of weak mineralization. The following list comprises all the waters, classed according to their composition: 1) calcium bicarbonate waters with mineralization up to 0.5 g, 2) calcium sulphate waters—from 0.5 to 5.0 g, 3) sodium sulphate waters 5.0—25.0 g, and 4) chloride waters with mineralization above 25 g. Many transitional forms are to be observed lying between these typical cases.

The waters of the lower zones—5 and 6—contain hydrogen sulphide, the maximum content of which amounts to 340.0 mg per litre. The temperature of the waters changes within the range of 2—6°C, it being 3—5°C in the lower zones. The average annual temperature for the town of Perm is +1.5° C.

The properties of the waters in the zone of argillaceous and striated dolomites undergo, just as in the other zones, a fairly marked change. Their mineralization fluctuates from 2 to 100 g per litre, and here we notice the striking fact of it growing regularly in the direction of the dip of the strata,

Composition of Waters of Gypsum and Striated Dolomites (in g per litre)

Date of sampling	Well No	Plot made bare, m	Distance from outcrop of rocks	Dry residue	Na	Ca	Mg	Cl	SO_4	HCO_3	H_2S
1937, III	2358	43—56	0.9	2.668	0.073	0.533	0.095	0.030	1.602	0.207	
VIII	2358	43—56	0.9	2.852	0.075	0.539	0.106	0.051	1.650	0.189	0.005
III	2359	50—56	1.7	5.188	0.799	0.541	0.154	0.962	2.153	0.159	not determ.
III	1738	48—56	1.9	10.496	2.491	0.468	0.231	2.068	4.507	0.159	
VIII	1738	48—56	1.9	10.360	2.484	0.468	0.292	2.108	4.477	0.164	0.018
Mean 10 for 1936—37	2200	59—69	2.5	18.648	5.212	0.426	0.348	5.283	5.867	0.390	
1937, III	2619	73—93	2.6	24.848	7.075	0.900	0.395	10.653	3.891	0.245	0.048
VIII	2611	55—69	2.5	29.180	8.962	0.975	0.358	13.801	3.563	0.265	not determ.
III	2351	55—77	2.6	40.928	10.500	1.553	1.239	19.837	3.479	0.245	0.043
VIII	2351	55—77	2.6	38.400	9.403	1.556	1.126	17.826	3.499	0.213	0.033
III	2618	56—74	3.4	43.760	11.351	1.752	1.032	21.240	3.065	0.191	
VIII	2318	56—74	3.4	40.424	10.922	1.630	0.886	19.647	3.377	0.196	0.070
III	2632	57—78	3.7	75.820	21.954	2.862	1.747	41.761	2.863	0.275	
VIII	2632	57—78	3.7	94.248	27.185	3.051	1.779	50.023	3.110	0.297	not determ.

which is very distinctly illustrated by the isolines of the mineralization of waters in this zone (Fig. 2). The character of the isolines plotted on a geological map testifies to a close relationship existing between the magnitude of the total mineralization of the waters and geological texture.

As to the composition of these waters, this stands in close dependence upon mineralization, wherefore the types of waters of this zone have a strict territorial distribution in the given zone (table).

Analysis of the curves plotted in Fig. 3 shows that the waters undergo a change from the calcium sulphate type with low mineralization, through the sodium sulphate type with middle mineralization, to the chloride type with high mineralization. The line representing the sum equivalents of sodium and potassium intersects the line of the chlorine-ion, while the line of the sum of calcium and magnesium intersects the line of the sulphate-ion. Their points of intersection correspond to a mineralization of 25—30 g. Waters of such a transitional composition containing alkaline metals almost equivalent to chlorine were found in argillaceous dolomites in wells 2611 and 2619.

Outstanding interest belongs here to the fact that the location of these wells on the site satisfies the condition of the distribution of waters in the thickness of argillaceous dolomites, in other words, the wells are situated between isolines 20 and 30 (Fig. 2).

A similar change in the composition of waters, as dependent upon mineralization, is observable throughout the whole bulk of the rocks of the district cross-sectioned in the vertical plane.

The phenomenon of the shift of waters is accounted for by physico-chemical processes occurring between the sedimentary rocks and the solutions migrating along the fissures. The water run off from the surface into a gypsum-bearing rock rapidly becomes saturated with gypsum, a fact which has been proved experimentally. Travelling further on through the rock, the water becomes enriched with sulphates, this being due to a reaction of exchange, in which the calcium of the solution displaces sodium, potassium and magnesium from the rock. With regard to gypsum, the solution remains all the time saturated, on account of the high content of the latter in the solid phase. Simultaneously with the aforesaid processes there takes place the leaching out of sodium chloride, which likewise increases the concentration.

The considerable accumulation of sodium in the solution causes a change in the reaction of exchange and in fact reverses it. Beginning from a concentration of sodium of about 0.25 g-equiv., which approximately corresponds to 25—30 g of salts per litre, the reaction ef exchange becomes reverse, namely: the sodium of the solution displaces calcium from the rock. From this moment on the content of the sulphate-ion tends to decrease which is recorded by a bend in the curve; as to the concentration of sodium, this, though it continues to increase, does so less intensely than the concentration of the chlorine-ion. After the sodium concentration has reached 0.3—0.4 g-equiv., there appear in water calcium and magnesium chlorides and the water gradually turns into brine.

The substitution of calcium for sodium involves deep changes in the composition of the waters. It is well known that the content of gypsum in solution is liable to decline with increase in the concentration of calcium ions ([2],[3]). The displacement of the calcium of the rock causes the sulphate to be thrown out of solution, the latter falling out in the form of gypsum, which contributes to a considerable accumulation in the solution of calcium and magnesium in the form of chlorides, whose content is found to be very great in brine. The analysis of this brine is as follows: $t°$—4.6°C; spec. grav.—1.093; pH—6.62; dry residue—143.52 g; Na—29.43; K—9.81; NH_4—0.03; Ca—

6.87; Mg—4.82; Sr—0.082; Cl—78.79; Br—0.15; I—0.017; SO_4—1.76; HCO_3—0.064; SiO_2—0.005; R_2O_3—0.012; Ra—$2.4 \cdot 10^{-11}$; H_2S—nil*.

The formation of brines, apart from the process of leaching and exchange reactions involving a rise in the concentration of salts in solution, is also likely to have been prompted by the phenomenon of wetting of the rock by water. The author has experimentally verified that fact that with sufficiently high concentrations of salts in water the rock, upon being wetted with brine, is apt to absorb more molecules of water and fewer molecules of salts than what could be expected from the respective proportions in the solution.

In our experiments on the consecutive leaching out of argillaceous dolomite and gypsum, we obtained waters with a content of dissolved matter equalling 26—27 g per litre. In the waters of the leaching out of argillaceous dolomite a great increase in the concentration of magnesium was observed (mineralization increased 8.7 times, while $MgCl_2$ rose 100-fold). Such a behaviour of magnesium is not to be explained by leaching alone; we have here an exchange of cations between the solution and the particles of argillaceous dolomite. This finds evidential support in the fact that in the waters of the leaching out of natural gypsum only the salts of $CaSO_4$ and NaCl were discovered.

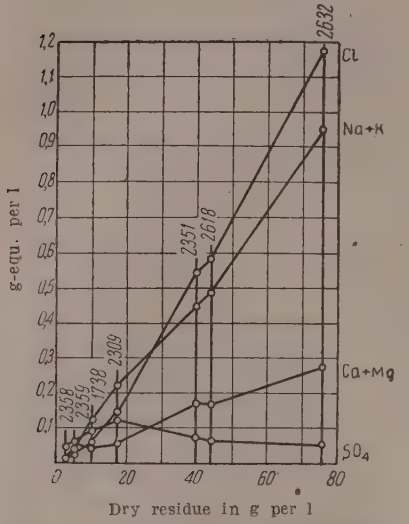

Fig. 3. Dependence of the composition of underground waters upon mineralization.

The subterranean waters of the oil deposits of California, described by Rodgers ([4]), whose properties depend upon their geographical location and the depth of the bed, provide, in our opinion, a striking example of the interaction of solutions and rocks; they indeed afford, as it appears to us, a reflection of a natural process, so far but little studied in hydrochemistry. Particular interest belongs to the availability here of waters of various types in the same level, which recalls to some extent the properties and character of the distribution of the waters of Levshino.

V. A. Sulin, while studying the subterranean waters of oil deposits within the western Urals, in the region of Verkhni Choussovsky Gorodki, arrived at the conclusion that the composition of the waters of the Perm sediments is, in point of fact, a further evolution of the chemical composition of the surface waters penetrating here, the process being due to the interaction of the dissolution of gypsum and rock salt ([5]).

All the waters of the zone of argillaceous dolomites contain hydrogen sulphide, whose presence does not, however, in any way characterize the type of the water. Without drawing a rash conclusion regarding the reason of the appearance of hydrogen sulphide in the waters of Levshino, we may yet take upon ourselves to say that its source lies in dolomites, from which it proceeds through desorption in course of the wetting of the rocks with water. The concentration of hydrogen sulphide in waters grows with the enhancement of mineralization, which fact may be indirectly indicative of the exi-

* Sample taken from bore-hole 2622, situated 360 m north from bore-hole 2618 (Fig. 2), from argillaceous dolomites and marls, at a depth of 41.0 m.

stence of a process of this desorption. The content of hydrogen sulphide in limestones and dolomites is, according to determinations made by Nenadkevich, constant, amounting to about 0.01% ([6]). In the opinion of W. I. Vernadsky, Member of the Academy, hydrogen sulphide in such a state is apt to keep for a sufficiently long time ([7]) its static condition being dependent upon the limits of its adsorption by the rock. The absence of any fossil fauna in the layers under consideration confirms the supposition that the deposition of sediments took place here in a medium of hydrogen sulphide, and this leads us to believe that hydrogen sulphide may have partly been conserved in the rocks up to the present time.

From the aforesaid it follows that water travelling along fissures not only enhances mineralization, but is also apt to change its composition on the way. Therefore, the existence of gypsum waters in the north of the district, that of sodium sulphate in the middle part, and that of chloride brines in the south of the same district should be understood as a historical path along which the formation and metamorphization of waters proceeded, while each type is to be considered as a step in the process of this evolution.

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GEOCHEMISTRY

MERCURY IN BARYTES

By A. A. SAUKOV

(Communicated by A. E. Fersman, Member of the Academy. 31.XII.1938)

What I am going to report here is an investigation of barytes from different deposits of the USSR for their mercury content. The samples were supplied by the Mineralogical Museum of the Academy of Sciences of the USSR, the Museum of the Moscow Geological Prospecting Institute (through N. A. Smolyaninov), as well as by a number of expedition workers, and taken from our own collections. To my knowledge, there are no references in literature in regard to occurrence of mercury in barytes. However, earlier investigations in this field had been carried out by K. A. Nenadkevich, who by an extremely sensitive spectral method of his own, used for determination of mercury, has for the first time established its presence in barytes and fluorites from certain deposits. It is to be regretted that the investigations by Nenadkevich have not been published as yet.

The two methods I have principally used in my investigation are the spectral analysis of Nenadkevich, which served the purpose of qualitative and rough quantitative estimation, and the nephelometric method, which is my own way to detect small quantities of mercury ([1]) for the quantitative analysis. With the principles of the former method I was kindly acquainted by the author himself, who has readily helped me in designing the necessary apparatus and otherwise assisted right along with consultation, and I am profiting of this occasion to express my deepest gratitude to him.

The method of Nenadkevich may be essentially described as follows: the material to be tested, in the amount of 1—3 g, reduced to powder, is placed in a quartz test-tube, connected by means of vacuum paste with a modified Geissler tube provided with two ground-in cocks and platinum electrodes, sealed into ground-in glass stoppers, which may be removed from the tube if necessary. The electrodes are enclosed in small aluminium cylinders, which are protecting the electrodes from mercury contamination, and in case of such contamination can be easily replaced by new ones. Before putting it into operation, there is created in the system by means of an oil-pump a vacuum of a few thousandths of a millimeter. The electrodes are connected to the terminals of a Ruhmkorff's coil fed by a current charged from an accumulator with tension of 4 V. The luminescence of the tube is observed with a visual spectroscope. If the tested substance in the quartz test-tube, connected to the vacuum system, is heated on the flame of a gas burner, the mercury vapours enter the tube and their spectra can be easily observed by means of a spectroscope. The green line with $\lambda = 5460.724$ is particularly well observable. It was proved by experiment that in the manner described mercury can rapidly be detected in a number of minerals and rocks, be it that its

amount is as low as one millionth of one per cent. Even a rough quantitative estimation of small quantities of mercury (ranging from $1 \cdot 10^{-6}$ up to $1 \cdot 10^{-3}$%) can be made from the intensity of the spectra by an experienced worker. A quantitative estimation of mercury in barytes has been effected according to the nephelometric method we have devised (1).

The results of both kinds of estimation are given in Table 1.

Table 1

Mercury Content in Some Barytes of the USSR

Deposits	Characteristic of barytes	Results of spectral analysis	Results of quantitative analysis, in %
Tyuya-Muyun, Main lode	Yellow-brownish	Very bright line	$1.9 \cdot 10^{-2}$
Do	Brown	Do	$1.7 \cdot 10^{-3}$
»	Baryte tuff	Bright line	—
»	Red	Do	$8.5 \cdot 10^{-4}$
»	Milk-coloured	Exceeding medium intensity	$5.6 \cdot 10^{-4}$
»	White, lamellar	Bright line	$6.4 \cdot 10^{-4}$
Tyuya-Muyn, Great baryte cave .	Zonal	Weak line	—
» » Academic lode . . .	—	Medium intensity	$5 \cdot 10^{-4}$
Ashat, Turkestan ridge	White, massive	Very bright line	$1 \cdot 10^{-3}$
Khaidarkan, North field	White	Do	$2 \cdot 10^{-3}$
Parvin deposits, Tadjik Aut. SSR	Optical	Medium intensity	$2.4 \cdot 10^{-5}$
Kulikolon, Tadjik Aut. SSR . . .	White, 1st generation	Very bright line	$1.5 \cdot 10^{-3}$
Do	Translucent, 2nd generation	Medium intensity	—
»	Transparent, 3rd generation	Exceeding medium intensity	$7.2 \cdot 10^{-4}$
Baryte mine, South Ural	Bluish-white	Medium intensity	$2.8 \cdot 10^{-4}$
Askyz region, West. Siberia . . .	—	Do	$1.5 \cdot 10^{-4}$
Novy Karantin, Crimea	Baryte with chalcedony	Nearly bright line	$3.4 \cdot 10^{-4}$
Mangyshlak, Kara-tau	Baryte with iron-ochres	Nearly medium intensity	—

A far greater number of barytes investigated have on spectral analysis shown faint lines of mercury or even none at all. Those were barytes from: Olenij Ostrov (Karelia), Slyudyanka (Transbaikal), Salagyr, Chuvash steppe, Kuznechikha (Chelyabinsk province), Koktebel, Katyrta-Sorai, Kerch and other ore-fields of Crimea, Sabregele and Lokkary (Georgia), Dzhezkazgan, Badama, Kzyl-espe, Naizatas, Maikain (Kazakhstan), Takeli and Ayubinskoye, Barnavadjskoye etc. (Tadjik Autonomous Socialist Soviet Republic), Arpaklaka (Turkmen Autonomous Socialist Soviet Republic).

The mercury content in them is less than $1 \cdot 10^{-5}$%.

Though no cinnabar inclusions whatever were observed under the microscope in barytes from mercury deposits of Khaidarkan and Ashata, they were for safety sake subjected to repeated treatment with aqua regia, after having been previously reduced to fine powder.

Still greater difficulties were experienced in removing mercury, in the manner described above, from other barytes, which has led to the suggestion that mercury is entering into the crystalline lattice of this mineral.

Taking into account the radii of ions, we have for Ba^{2+}—1.43Å and for Hg^{2+} —1.12 Å. For univalent mercury the ion radius is indeed unknown, but we may derive its approximate magnitude from the magnitudes of neighbouring ions: gold (1.37) and tallium (1.49). The mean obtained for Hg^{1+}

is 1.43 Å, i. e. the same as for barium. Consequently, if one is to judge by the magnitude of ion radii, the suggestion that barium is isomorphic with univalent mercury is gaining more ground.

In order to verify this assumption, we have carried out, in collaboration first with N. S. Krupenko and then with N. H. Aydinyan, experiments on the synthesis of barium sulphate from solutions in the presence of salts of univalent and bivalent mercury. In the former case $BaSO_4$ precipitated from the solution $Ba(NO_3)_2$ in the presence of $HgNO_3$ through the agency of Na_2SO_4; in the latter case precipitation $BaSO_4$ was effected from the solution of $BaCl_2$ in the presence of $HgCl_2$ by means of the same agent. In both cases the precipitates were very thoroughly washed, several times treated with aqua regia, as well as with nitric acid, upon being heated and cooled, in order to remove all the mercury, which had escaped the crystalline lattice.

Ultimately, upon all the tests with bivalent mercury, it became evident that the baryte derived has no mercury content whatever. This has been confirmed both by the spectroscopic method of K. A. Nenadkevich and our own nephelometric method.

On the other hand, univalent mercury has been steadily and in rather considerable quantities entering into the composition of synthetic baryte, the quantity each time depending upon conditions and velocity of precipitation. K. A. Nenadkevich arrived at essentially like conclusions.

The results of our tests are given in Table 2.

Table 2

Date of test	Synthetic $BaSO_4$ in the presence of:	Results of spectral analysis on mercury	Results of quantitative analysis on mercury, in %
25.X.1937	Hg^{1+}	Very bright line	0.0027
	Hg^{2+}	Weak line	not found
4.I.1938	Hg^{1+}	Very bright line	0.065
25.XI.1938	Hg^{2+}	Weak line	not found
	Hg^{1+}	Very bright line	0.51

The following conclusions may be drawn from the above investigations:

1. Univalent mercury may be an isomorphic component of barium sulphate.

2. The foregoing does not apply to bivalent mercury.

3. If the formation of natural barytes is proceeding in the presence of mercurous oxide, a «capture» of mercury by the baryte lattice is taking place. Evidently, the greater the quantity of mercury, the more of it (up to a certain limit) would be captured upon by the baryte. Therefore, barytes occurring near mercury deposits, and in general in regions with a high mercury clark, would be enriched by mercury, and consequently, might occasionally serve as prospects in the search of mercury deposits. This can be exemplified by a number of barytes of Middle Asia.

4. The extremely high mercury content in some barytes of Tyuya-Muyun is partly explained, perhaps, by the reason indicated above, though the possibility of other explanations is not excluded, one of them being the presence of rich radioactive ores at that place.

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MINERALOGY

NEW DATA ON LINARITE IN ORE-DEPOSITS OF KAZAKHSTAN

By F. V. CHUKHROV

(Communicated by A. E. Fersman, Member of the Academy, 31.XII.1938)

Linarite is commonly classified as one of the rare products of hypergenesis, which may be due to a certain extent to the possibility of its being identified with other minerals. Reliable data on linarite are available in literature only in connexion with three localities in the Kazakhstan steppe, namely: Kara-Oba, Bistcheku and Kyzyl-espe. Description of linarite from the former two deposits is given by P. W. Jeremeev ([1],[2],[3]). Linarite from Kyzyl-espe has been characterized by V. I. Krzyžanovsky ([5]). In view of the wide range of distribution of copper-lead ore-deposits in Kazakhstan, it was natural to assume that linarite is a mineral far more widespread over there. This assumption was fully confirmed by the mineralogical investigations in the hypergenesis zone of the Kazakhstan ore-deposits, carried on by the author as member of the Central Kazakhstan Expedition under the auspices of the Academy of Sciences. During these investigations, caledonite was found for the first time in Kazakhstan, hitherto having been known in the USSR but from the Berezovsk beds in the Urals ([4]). The present note contains a brief characteristic of new finds of linarite. This mineral has been recognized in the following localities, of which the former three are situated in the Kazakhstan steppe region, and the latter—in South Altai:

1) Dzhezkazgan (Karsakpai district, Karaganda province);
2) Azhim (Amangeldin district, Kustanai province);
3) Berkar (Karkaralin district, Karaganda province);
4) Manka (Zaysan district, East Kazakhstan province).

In Dzhezkazgan linarite has been discovered in the Raymund ore district (Raymund) and in the eastern part of the Krestovozdvijensk ore district (Kresto-vostok). In the Raymund district linarite has been identified in the form of crusts on beaveryte and ore-bearing sandstone, occasionally in stuf paragenesis with chrysocolla and atacamite. In the district of Krestovozdvijensk linarite partly occurs in the form of veinlets (6—8 mm) in sandstone, made up of elongated grains orientated to the walls in various directions, sometimes arranged in radial concretions with distinct cleavage. In shallow cavities, among linarite aggregations, are observed vaguely shaped lamellar crystals of the mineral up to 0.1 mm thick. Apart from the veinlets, linarite occurs in the form of thin minute crystalline crusts on the planes of sandstone, and partly on beaveryte as well. To a great extent, the linarite of the Krestovozdvijensk region gave way to antlerite which, as indicated above ([6]), is a mineral relatively widespread over there. Besides

beaveryte and antlerite, in association with linarite have been found: malachite, azurite, cerussite, gypsum etc.

The following table shows the results of an analysis of linarite from the Krestovozdvijensk region of Dzhezkazgan. The analysis was made on linarite, filling the v einlet in the sandstone, by the analyst V. M. Senderova.

Components	Data of analysis in %	Molecular quantities	Molecular ratios
PbO	47.15	0.2112	2.00
CuO	26.43	0.3321	
SO_3	20.70	0.2586	1.00
H^2O	5.00	0.2777	1.07
Insoluble residue (in HNO_3)	1.22	—	—
	100.50	—	—

The analytical data may be well represented by the formula: (Cu, Pb) $SO_4(Cu,Pb)(OH)_2$. The insoluble residue is presented by quartz particles.

In the ore field Azhim, situated 185 km north-west of Dzhezkazgan, linarite is relatively not rare, though it occurs in very small quantities. It usually segregates in the form of minute crusts on quartz, which in this case takes place of the matrix. The Azhim deposits count among their typical hypergene minerals cerussite, jarosite and native sulphur.

Linarite has a comparatively wide range of distribution in the Berkar ore field, where it is represented primarily by minute crystalline crusts in the cracks of ores from the zone of hypergenesis. In stuf paragenesis with linarite have been met: malachite, azurite, lead-ochre and others. It is noteworthy that part of the lead-ochre in Berkar, as well as in Dzhezkazgan, is represented by beaveryte.

In the Manka deposits linarite has been identified in the shape of elongated, imperfectly shaped crystals and their aggregations, partly of radial structure, on gangue quartz and dark grey cerussite. In association with linarite, apart from cerussite, have been recognized: caledonite, native gold etc.

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MINERALOGY

CHKALOVITE

By V. I. GERASIMOVSKY

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In the summer of 1936, in the south-eastern part of the Lovozero alkaline massifs (Punkaruayv mountain), situated in the central part of the Kola peninsula, the author found a mineral, by its external appearance somewhat resembling natrolite, though differing from the latter by a less distinct cleavage. This circumstance prompted the author to take several samples containing the discovered mineral for the purpose of investigation. When the mineral was studied, it became evident that by its properties it could not be identified with minerals already known, and was therefore named chkalovite, in honour of the great aviator of our times, Valery Pavlovich Chkalov who was the first to accomplish in 1936 the non-stop flight from Moscow, via north pole, to the United States of America.

Physical Properties. Chkalovite has not been encountered in crystalline form. It occurs in the form of grains, with dimensions $5\times4\times3$, $7\times4\times3$ cm. The mineral is of white colour. Lustre vitreous. Semi-transparent. Cleavage, as examined with the microscope, occurs in one direction, is of fair quality, and usually observed with difficulty. In one direction parting plane poorly expressed. Fracture from uneven to conchoidal. Hardness$\sim$6. Specific gravity determined by a pycnometer equal to 2.662.

In the blowpipe flame easily fused to a transparent colourless globule. With borax and phosphate gives a colourless bead both in oxidating and reducing flames.

Optical Properties. Chkalovite in thin-section is colourless. Optically biaxial, positive. $2V=78°$ (measurement taken on the Fedorov stage). Refraction indices (determined by the immersion method): $N_g=1.549$ and $N_p=1.544$. The plane of optic axes coincides with the cleavage plane, macroscopically observed, the latter being well pronounced in thin-sections. Apart from that, the thin-sections exhibit poor cleavage too, usually in one direction, rarely in two (in sections perpendicular to one of the optic axes). Measurement of the imperfect cleavage in parallel direction, in the same thin-section, yields data of wide divergence from one another. This is explained by the fact that the given cleavage is poorly expressed and is therefore determinable by measurement but with great difficulty.

By comparing the results of measuring in various thin-sections of imperfect cleavage in one and the same direction, as well as in different directions, more or less similar data are derived, namely, on the average:

$$P \text{ (cleavage pole)} \begin{cases} N_g = 51° \\ N_m = 54°30' \\ N_p = 60° \end{cases}$$

It is hereby necessary to note that there are very considerable deviations between separate measurements of cleavage in relation to N_g, N_m and N_p: N_g ranging from 46 to 58°, N_m=50–59° and N_p=53—66°. It is probably the identical poorly expressed cleavage observed in different thin-sections. The angle between the different directions of the indistinct cleavage usually ranges from 68° to 76°, the average being~72 . In one of the thin-sections (in the section of the acute bisectrix) imperfect cleavage of different orientation has been found, namely:

$$P \angle \begin{matrix} N_g = 51° \\ N_m = 89° \\ N_p = 39° \end{matrix}$$

It forms a right angle with good cleavage and presents cleavage along the prism, if the mineral is to be considered orthorhombic, or along the dome, if it be monoclinic. In thin-sections parallel to the parting plane is observed the emergence of the optic axes of the ellipsoid.

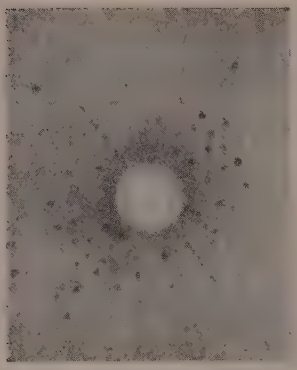

Fig. 1. Lauegram of chkalovite.

By its optical properties chkalovite may be identified as a mineral of the orthorhombic or monoclinic system. The optical data are inadequate for an ultimate solution of the question, as measurement on the Fedorov stage of the imperfect cleavage has shown great divergences between separate measurements of the same as well as of different directions of cleavage. Hence it is impossible to judge about the symmetrical arrangement of the imperfect cleavage in relation to the optical orientation of the ellipsoid, while deciding on the question of the syngony of chkalovite.

X-Ray Investigation. On the basis of the optical data it was impossible to determine to which one of the two systems—the orthorhombic or monoclinic—chkalovite could be referred, and therefore, in order to solve the problem, it became necessary to make an X-ray study of the mineral.

As chkalovite is unknown in crystalline form, it was necessary while investigating it by Laue's method to use a small grain bounded on one side by a distinct cleavage surface. During the X-ray procedure the aforesaid grain was oriented in such manner as to have the X-rays fall perpendicular to the cleavage surface. The X-ray picture was taken in the Roentgeno-Structural Laboratory of the Physical Department of the Academy of Sciences of the USSR. The X-ray procedure: rays Fe—series $K_\alpha \approx 1.93$ Å, tension 40 kV, current intensity 12 mA, duration of the exposure 2 hours, and the distance of the object from the film 15 mm.

On the basis of the obtained Lauegram of reflected rays, N. V. Belov (the Head of the Roentgeno-Structural Laboratory) considers chkalovite a mineral of orthorhombic syngony (Fig. 1).

Simultaneously with the Lauegram a powdergram was taken at the above-mentioned laboratory (Fig. 2). X-ray procedure: rays Fe—series $K_\alpha \approx 193$ Å, Evansen chambers D=57.37 mm, column diameter 0.75 mm, tension 40 kV, current intensity 12 mA, and duration of exposure 5 hours.

Table 1 shows dimensions of the interspacings for the reflecting chkalovite planes. They are given in case it would be possible to draw a comparison

between chkalovite and minerals of analogous or similar chemical composition, which might be found elsewhere.

Table 1

Plane Spacings for Chkalovite

Dhkl in Å	Intensity of lines	Dhkl in Å	Intensity of lines
5.21	Medium	1.40	Strong
3.97	Very strong	1.38	»
3.27	Strong	1.33	Medium
2.74	»	1.31	»
2.46	Very strong	1.30	Very weak
2.41	» »	1.28	Weak
2.30	Medium	1.27	»
2.20	»	1.22	Very strong
2.02	»	1.17	Medium
1.99	»	1.16	»
1.79	Strong	1.15	Weak
1.75	Weak	1.12	Medium
1.71	Medium	1.11	Strong
1.69	»	1.09	Very strong
1.66	»	1.08	Weak
1.60	»	1.06	Medium
1.56	»	1.033	»
1,51	»	1.026	»
1.48	Strong	1.014	»

Chemical Properties. In diluted and strong acids HCl and NHO_3 chkalovite dissolves easily and without being heated, with separation of flaky silicic acid. In H_2SO_4 it dissolves with great difficulty.

The chemical analysis of chkalovite, performed by Pereverzeva (Geol. Inst. of Acad. of Sc. of the USSR) is given in Table 2. The latter contains

Fig. 2. Powdergram of chkalovite.

(3rd column) quantitative determinations of Al and Be in chkalovite, effected by J. S. Nesterova (Geol. Inst. of Acad. of Sc. of the USSR).

The formula of chkalovite, according to the chemical analysis, may be represented as follows: $Na_2O.BeO.2SiO_4$ or $Na_2Be(SiO_3)_2$. Thus, chkalovite is to be regarded as a metasilicate. The group of metasilicates comprises the following beryllium minerals: leukophanite, meliphanite, beryl, which, as known, differ very much by their chemical composition from chlakovite.

By its chemical composition the latter stands nearest to such minerals as eudidimite and epididimite (Table 2), occurring in small quantities in nepheline syenites on the Arö island, Langezundfjord, Norway, and at Narsarsuk, Greenland.

A spectroscopic study of chkalovite carried out by S. A. Borovick (Geol. Inst. of the Acad. of Sc. of the USSR) has detected, in addition to the compounds revealed by chemical analysis, but Mn (a very weak line).

Occurrence and Parageneses. In Lovozero Tundras chkalovite has been found but on the eastern slope of the Maly Punkaruayv mountain in two ussingite deposits, one of them representing two pegmatite

Chemical Analyses of Chkalovite, Leucophanite, Meliphanite, Epididimite and Eudidimite

Table 2

Oxides	Chkalovite		Chkalovite	Leucophanite	Meliphanite	Epididimite	Eudidimite	
	%	equiv. quant.	%	%[1]	%[2]	%[3]	%[4]	%[5]
SiO_2	53.81	0.946	—	48.59	43.60	73.74	72.19	73.11
TiO_2	none	—	—	—	—	—	—	—
ZrO_2	none	—	—	—	—	—	—	—
Al_2O_3	none	—	—	—	—	—	—	—
Fe_2O_3	0.30	0.002	0.75	0.45	4.61	—	—	—
FeO	0.12	0.002	—	—	—	—	—	—
BeO	12.67	0.506	—	—	—	—	—	—
BaO	none	—	11.28	10.03	10.80	10.56	11.15	10.62
MnO	none	—	—	—	—	—	—	—
MgO	none	—	—	—	—	—	—	—
CaO	0.37	0.07	—	0.27	0.16	—	—	—
SrO	none	—	—	22.94	29.56	—	—	—
Na_2O	28.93	0.459	—	—	—	—	—	—
K_2O	0.13	0.010	—	12.42	7.98	12.48	12.66	12.24
H_2O, + 110°	none	—	—	—	0.23	—	—	—
H_2O, — 110°	0.23	0.013	—	} 1.08	—	} 3.73	3.84	3.79
F	none	—	—	5.04	5.43	—	—	—
Cl	none	—	—	—	—	—	—	—
SO_3	0.22	—	—	—	—	—	—	—
	99.78	—	—	101.63	101.37	100.91	99.84	99.76
$-O=Cl_2$	—	—	—	2.48	2.29	—	—	—
Total	99.78	—	—	99.15	99.08	100.91	99.84	99.76

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veinlets in the form of expansions 30 and 50 cm thick, with visible length of 2—3 m, confined to the vertical fissures of plane partings, among eudialyte «Schlieren». The latter are lying among foyaite with sodalite. Pegmatite veinlets are packed in ussingite with a high content of natrolite. Both of these minerals were developed upon sodalite met at the present time in subordinate quantities. Of accessory minerals there occur: schizolite, marmanite, sphalerite, sometimes in considerable quantity, and insignificant quantities of steenstrupine, neptunite, erikite and individual deposits of galenite. Along the edges of the ussingite dilations are encountered: nepheline, microcline, aegirite, eudialyte and occasionally ramsayite as well.

The second deposit presents a segregation of an irregular form in the shape of «Schlieren», with dimensions $1 \times 0.6 \times 0.4$ m, lying in the midst of foyaite with sodalite, the bulk of it being enclosed in ussingite. It contains the following accessory minerals: microcline, sodalite, eudialyte, schizolite, tainiolite(?), neptunite, ceolite, with isolated occurrence of sphalerite, steenstrupine and erikite.

In both of the aforementioned deposits chkalovite occurs in small quantities having developed there prior to ussingite, natrolite, sphalerite, galenite and neptunite. Its age correlation with other minerals is unclear, as it does not occur in immediate contact with them.

Conclusions

Among the minerals of Lovozero alkaline massif chkalovite is the only one independent beryllium mineral. In other minerals the presence of beryllium was not determined chemically, but by means of spectral analysis. By the latter method it has been detected in 21 minerals: microcline, orthoclase, nepheline, sodalite, ussingite, natrolite, aegirite, arfvedsonite, schizolite, steenstrupine, tainiolite(?), eudialyte, catapleiit, sphene, astrophyllite, chinglusuite, novoelpidite, lovenite, neptunite, fluorite and galenite.

When studying chkalovite, the question arose as to what were the conditions which favoured the formation of an independent beryllium mineral—chkalovite, and why beryllium entering into its composition had not been dispersed among other minerals in the ussingite deposits, especially since chkalovite is contained in the latter in very small quantity. For that purpose 8 minerals were taken from the ussingite deposits containing chkalovite (sodalite, ussingite, natrolite, microcline, tainiolite(?), schizolite, steenstrupine and galenite) and subjected to spectral analysis for determination of the beryllium content. All of the enumerated minerals contained beryllium, whereby minerals (ussingite, natrolite and sodalite) from the ussingite deposits showed a high beryllium content—0.1—0.001 of one per cent (according to data of S. A. Borovick). It should be noted here that the spectral analysis of sodalite and natrolite from other places of the Lovozero massif points to a lower belyllium content therein. Consequently, the conditions under which the formation of ussingite had taken place, had been apparently more favourable to a higher beryllium concentration than had been the case at other places of the Lovozero alkaline massif. Therefore in this case the surplus of beryllium in excess of its ordinary concentration caused part of Be to segregate toward formation of an independent beryllium mineral. Hence the question arises whether beryllium, which by its physical and chemical properties is closely allied to aluminium, takes the place of the latter; in which case chkalovite ($Na_2OBeO.2SiO_2$) can be regarded as nepheline ($Na_2O.Al_2O_3.2SiO_2$) wherein aluminium had given way to beryllium.

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SOIL SCIENCE

ON THE METAMORPHISM OF PHOSPHORUS COMPOUNDS IN SOIL FORMATION IN THE HUMID SUBTROPICS OF TALYSH

By A. A. ZAVALISHIN and K. V. VERIGINA

(Communicated by L. I. Prassolov, Member of the Academy, 2.XII.1938)

In studying the peculiar subtropical soils of the lower belt of Mountainous Talysh and the Caspian accumulative formation plain within the boundaries of the Lenkoran region of the Azerbaidjan SSR in connexion with working out a land-betterment scheme for the tea culture, we succeeded in procuring a number of data characteristic of the processes of absorption and accumulation of phosphorus in course of weathering and soil formation under various conditions.

It is possible, to a certain extent, to judge of the metamorphism of phosphorus compounds by changes in the solubility of P in extractions from the soil by solutions with different pH. The following varieties of soils were taken for investigation: slightly podzolized yellow soil on an alluvium of feldspar sandstone under an oak-hornbeam forest from a declivitous hill slope (No 1).

Podzol-like soil with surface water lodging characteristics on heavy, dense, dark-brown clay under an oak forest intergrown with *Parotia persica* from the upper part of a marine terrace in an accumulative formation plain (No 2).

Gley-like soil with slight podzolization characteristics on a clay-loam subsoil under an oak forest from a second marine terrace in an accumulative formation plain (No 3).

Silt-gley (marsh) soil on heavy clay under an oak forest intergrown with bearded alder on marshland on the lower lowlying part of a second marine terrace in an accumulative formation plain (No 4).

The results of the determination of the gross contents of phosphorus, of its solubility in a hydrochloric acid and an alkaline extraction and the reaction of an aqueous suspension of the mentioned soils are given in the table.

The distribution of gross phosphorus according to the profile of the soils investigated shows clearly the biological accumulation of this element in the upper horizons. Accumulation checks to the greatest extent the absorption of phosphorus in slightly podzolized yellow soil and in podzol-like soil, but in the former the content of phosphorus in the accumulation horizon is nevertheless less than in the mother formation. Thus, in this soil the accumulation of phosphorus in the upper horizon proceeds only to a certain extent and fails to compensate for the absorption of P occurring in course of the weathering of the mother formation.

In the soils, whose evolution has not progressed to such an extent, and

Contents of Phosphorus and Its Solubility in Acid and Alkaline Extractions after the Profile of Soils of the Lenkoran Region

Description of soil	Depth of sampling in cm	Humus in %	pH of aqueous suspension	Gross P_2O_5 in mg per 100 g soil	Soluble P_2O_5 in mg per 100 g soil			% of soluble P_2O_5 to gross amount	
					K_2CO_3 (0.2 *n*)		HCl (0.2 *n*)	K_2CO_3	HCl
					Miner.	Organ.			
No 1 Slightly podzolized yellow soil	0— 5	9.7	6.6	130	7.5	traces	1.2	5.8	1.0
	10—15	2.6	6.0	120	3.7	»	1.2	3.1	1.0
	20—25	1.7	5.5	70	2.5	»	1.2	3.6	1.8
	35—40	0.8	5.1	40	1.2	»	1.2	3.1	3.1
	50—55	0.4	5.3	40	1.0	not determ.	1.9	2.5	4.7
	65—70	0.4	5.3	30	traces	» »	1.9	traces	6.2
Mother formation . . .	115—120	none	7.4	360	1.0	» »	155.0	0.2	43.0
No 2 Podzol-like on dense clay	0— 5	5.9	6.5	110	10.0	traces	6.2	9.1	5.7
	10—15	1.5	6.5	80	5.0	»	2.5	6.3	3.1
	20—25	0.9	6.3	50	2.5	»	1.2	5.0	2.5
	35—40	0.8	6.2	60	2.5	not determ.	1.9	4.1	3.1
	45—50	not determ.	6.0	30	1.7	» »	2.5	5.7	8.3
	55—60	» »	5.9	30	traces	» »	2.5	traces	8.3
No 3 Gley with characteristics of slight podzolization	0— 5	—	5.6	340	13.3	1.2	15.0	4.0	4.5
	15—20	—	6.1	170	4.2	traces	10.0	2.4	5.7
	37—42	—	6.2	180	1.7	none	11.2	0.9	6.4
	60—65	—	6.6	140	1.7	»	9.0	1.1	6.5
No 4 Silt-gley (marsh)	0— 5	13.4	6.2	380	10.9	0.8	30.0	4.0	7.9
	20—25	3.4	7.2	190	0.9	0.8	17.4	0.9	9.2
	35—40	2.2	8.2	200	0.8	traces	50.0	0.5	25.4
	75—80	1.2	8.5	220	0.8	none	62.5	0.4	28.9

Note. The solution of 0.2 *n* K_2CO_3 has a pH of about 12.0, the solution of 0.2 *n* HCl has a pH of about 0.7. The final pH of acid extractions differs from the initial value by a magnitude of from 0,1 to 0.2 pH. In extractions the ratio of soil to solution is 1 : 5.

which have been formed on comparatively young and little weathered sediments, i. e. in soils No 3 and 4 of the second terrace of the accumulation plain, the absorption of phosphorus is hardly expressed at all, wherefore its absolute content in accumulation horizons is apt to increase, while the extent of accumulation has a tendency to decline.

In all the profiles investigated the solubility of P_2O_5 in alkaline extraction reaches its maximum near the surface and tends to fall with greater depth, approaching zero at a depth of about 50 cm. In this way, the forms of phosphorus soluble in alkalies are being stored up only in the accumulation level of soils and are almost absent in the mother formation.

According to existing data on the solubility of various phosphates, it is known that if an extraction with a pH of about 12.0 be available, it will take up the soil phosphates bound up with iron and aluminium and also a part of the phosphorus contained in humus matter. However, the amount of the latter was found to be very insignificant in the soils investigated, and, consequently, the increased solubility of phosphorus in alkaline extractions from the accumulation horizons of soils can mainly be accounted for by their storing up the phosphates of sesquioxides as a result of the biological cycle of soil-formation.

The extent of the accumulation of the phosphates of sesquioxides in the upper levels of the soils investigated increases but slightly as the total content of phosphorus in the underlying horizons augments and depends but comparatively little upon the rate of podzol-formation.

Altogether different values regarding the solubility of phosphorus in different soil-profiles are obtained from hydrochloric acid extractions. In slightly podzolized yellow soil the solubility of phosphorus in a hydrochloric acid extraction taken from the upper horizons was found to be negligible. An inconsiderable increase of both absolute and relative solubility of P_2O_5 is observed with increased depth. In the mother formation the solubility of P_2O_5 in hydrochloric acid increases tremendously.

In other soils a certain increase of acid-soluble forms of phosphorus in the accumulation horizons is observable, this being the greater, the higher is the total content of P_2O_5 in the soil. On the whole it may be said that a definite inversely proportional dependence is apt to exist between the extent of podzolization of the soils and the accumulation of acid-soluble forms in humus-bearing horizons.

Thus, the forms of P rather easily soluble in hydrochloric acid and prevailing in the mother formations (in all probability mainly apatites) are subject to intense absorption in the process of weathering. The biological cycle of soil formation checks the absorption of phosphorus, as, owing to this cycle, P passes into such a form which under natural conditions is comparatively inert, namely the form of alumo- and ferri-phosphates. As the podzol-forming process weakens down, coupled with the accumulation of alkali-soluble forms of P and the general enhanced contents of that element in the accumulation horizons, we also find an increased solubility of phosphates in hydrochloric acid.

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GENETICS

CYTOGENETIC DETERMINATION OF A NEW AUSTRALIAN TOBACCO SPECIES AND ITS MODE OF ORIGIN

By DONTCHO KOSTOFF

(Communicated by N. I. Vavilov, Member of the Academy, 3.XII.1938)

In 1936 I received seeds from Dr. H. Wenholz, Australia, of Australian tobacco species. One package was labelled as «Wild tobacco, *Nicotiana* sp., coll. R. O. May, Bathurst, N. S. W., immune or highly resistant to blue mould *Perenospora tabacina*». In the list of tobacco species sent kindly to me by Dr. Roy E. Clausen, California, *Nicotiana suaveolens* from Bathurst with doubled chromosome number ($n=32$) was included. The plants that grew from the seeds sent to me by Dr. Wenholz were in many respects similar to *N. suaveolens* Lehm. and fit to the description given by Wheeler([3]) for the tetraploid *N. suaveolens* from Bathurst. They had 64 somatic and 32 gametic chromosomes, which means that they were tetraploids in respect to the Australian species having $n=16$. A large number of Australian tobacco species, as for example, *N. suaveolens*, *N. maritima*, *N. velutina*, *N. excelsior*, etc. have 16 gametic and 32 somatic chromosomes.

Tetraploid *Nicotiana* species from Bathurst grew very vigorously. Under favourable conditions it reached in height about 180 cm, while *N. suaveolens* Lehm. under the same conditions reached a height of ca. 70—90 cm. The flowers of the tetraploid species were in many respects like those of *N. suaveolens*, the leaves were, however, much larger, somewhat broader and with uneven surface. The flowers of *N. suaveolens* have a very agreeable odour, while those of the tetraploid *Nicotiana* smell disagreeably like the flowers of *N. maritima*, though not as sharp as those of the latter species. The capsules of the tetraploid species are similar to those of the F_1 hybrids *N. maritima*×*N. suaveolens*.

In studying the meiosis of the tetraploid form I found usually 32 bivalents, though occasionally one quadrivalent or one trivalent and one univalent were also found. It was of importance, from evolutionary point of view, to determine the mode of origin of this tetraploid form.

If we provisionally assume the number 16 for basic chromosome number for the *suaveolens—maritima—velutina* group of the Australian tobacco species, our tetraploid *Nicotiana* has then the genom formula *SS SxSx*. The fact that quadrivalents and trivalents were occasionally found during the meiosis of the tetraploid *Nicotiana* from Bathurst suggested that certain chromosomes of *S* genom are homologous or have homologous segments with the chromosome of *Sx* genom. In order to determine, however, more definitely the degree of homology of *S* and *Sx* genoms, we crossed the tetraploid

Nicotiana from Bathurst with the American tobacco *N. Sanderae* ($n=9$, $2n=18$). The hybrids between *N. Sanderae* and the Australian species are usually characterized by asyndetic meiosis. During the first meiosis in these hybrids usually 25 univalents were found, though one or two (rarely) bivalents sometimes also occur. The F_1 hybrids *Nicotiana* from Bathurst $\times$ *N. Sanderae* usually formed 16 bivalents and 9 univalents, which shows that *S* genom is homologous to *Sx* genom, since the chromosomes of these two genoms conjugated (autosyndesis) forming 16 bivalents, the nine univalents being obviously *N. Sanderae* chromosomes (figure).

I attempted further to investigate the degree of differentiation of genom *S* in respect to genom *Sx*. Autosyndesis in F_1 *Nicotiana* from Bathurst $\times$ *N. Sanderae* showed that *S* genom is closely related to *Sx*, but these observations were not sufficient to answer the question as to whether *Nicotiana* from Bathurst is an autotetraploid or else it is an allotetraploid from very closely related forms, the chromosomes of which conjugate during the first meiosis in their F_1-hybrids. In studying the procedure of the first and second meiotic anaphases and telophases in F_1-hybrids between tetraploid *Nico-*

The chromosomes from a pollen mother cell during the first meiotic metaphase separately drawn from the species hybrid *Nicotiana Eastii* ($n=32$) $\times$ *N. Sanderae* ($n=9$); sixteen autosyndetic bivalents of *N. Eastii* and nine univalents of *N. Sanderae*.

tiana and *N. Sanderae* I found that in some cells chromatin bridges were formed. The formation of chromatin bridges during the meiotic anaphases suggested that the chromosomes of *S* genom are structurally not identical in respect to the chromosomes of genom *Sx*, the structural differences being of the type of inversions. This statement, although being of importance from a cytogenetic point of view, does not decide the question of the autopolyploid or allopolyploid origin of the tetraploid form, since the structural changes that characterize *S* and *Sx* genoms might as well have taken place before as after the chromosome doubling. The morphological and the cytological behaviour of the tetraploid species from Bathurst, the Australian species of the group *N. suaveolens—maritima—velutina* and the species hybrids *N. maritima* $\times$ *N. suaveolens*, *Nicotiana* from Bathurst $\times$ *N. suaveolens* and *N. suaveolens* intraspecific hybrids, suggested that the tetraploid *Nicotiana* from Bathurst has originated by chromosome doubling in F_1 *N. maritima* $\times$ *N. suaveolens* hybrid or in a *N. maritima—N. suaveolens* segregate. It should be mentioned here that F_1-hybrids *N. maritima* ($n=16$) $\times$ *N. suaveolens* ($n=16$) have almost normal meiosis, forming usually 16 bivalents, and are fully fertile. Morphological appearance of the F_1 *N. maritima* $\times$ *suaveolens* hybrids is, in many respects, similar to that of the tetraploid *Nicotiana* from Bathurst. Disagreeable odour of the flowers of *N. maritima* dominates in F_1-hybrids which accounts for the presence of the same character in the tetraploid species. The odour of *N. maritima* flowers is, however, much more disagreeable than that of the tetraploid *Nicotiana* from Bathurst and of the F_1 *maritima—suaveolens* hybrids. *N. suaveolens* is resistant to most of the virus diseases our tobacco species are liable to in Moscow, while *N. maritima* is very susceptible to them; the tetraploid *Nicotiana* and the F_1-hybrids *N. maritima* $\times$ *N. suaveolens* are less resistant than *N. suaveolens*, but less susceptible than *N. maritima*. The flower shape of the tetraploid *Nicotiana* is similar to, though it is not identical with that of the flowers of the F_1-hybrids *N. maritima* $\times$ *suaveolens*. Such deviations, however,

are to be expected for the following three reasons: 1) They can be due to crossing-over and further to segregations in the allopolyploid *N. maritima—suaveolens*, since allopolyploids originating from F_1-hybrids with allosyndetic meiosis do always segregate[1]. 2) Chromosome doubling in plants ultimately leads to a series of changes in all the plant organs including the flowers[2]. 3) Accumulations of new hereditary changes in the newly originated tetraploid and eliminations of the less fitting forms by natural selection. I shall also mention here that the areals of distribution of *N. suaveolens* Lehm. and *N. maritima* Wheel. overlap each other in South Australia.

The hybrids between the tetraploid *Nicotiana* from Bathurst and *N. suaveolens* Lehm. are highly sterile, being «triploids», and only occasionally do they set very few seeds. The morphological appearance of the tetraploid *Nicotiana* from Bathurst, its physiological characters and cytogenetic behaviour give us sufficient ground to separate it into a species nova, which has originated by chromosome duplication in a hybrid and we shall call it *Nicotiana Eastii**. Cytogenetically it is more isolated from *N. suaveolens* and *N. maritima*, than the latter two species amongst each other, since the species *N. suaveolens* and *N. maritima* cross easily and their hybrids are fully fertile, while *N. Eastii* does not cross so readily either with *N. maritima* or *N. suaveolens* and its hybrids with these two species are highly sterile.

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* In the memory of Prof. E. M. East (1879—1938), who worked for 25 years upon the genetics of the genus *Nicotiana*.

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GENETICS

CYTOLOGICAL MECHANISM OF MOSAICISM AND OF CHROMOSOME REARRANGEMENT

By A. A. PROKOFIEVA-BELGOVSKAYA

(Communicated by N. I. Vavilov, Member of the Academy, 4.I.1939)

The studies of mitotic and salivary gland chromosomes of *Drosophila melanogaster*([5],[8]) have led to the conclusion that their heterochromatic regions correspond to the regions known as genetically inert. The investigations of mosaic strains of *D. melanogaster* clearly show that inert regions exert a profound influence on the characters depending upon the chromosome sections approximated to the inert regions as a consequence of some chromosome rearrangement. As we have now established, the modification of characters of the flies in mosaic strains is followed by a cytologically visible modification of the structure and behaviour of a given active chromosome section, as well as of its chemical nature.

The following phenomena have been established in the process of investigating chromosomes with the inert regions approximated to the *y-ac* loci (the sc^8 strain) or to the locus of white (w^{m4}, w^{m5} and $w^{m\,Med}$ strains):

1. The conjugation of chromonemata in the active section in question is highly weakened, this leading to a disturbance of its discoidal structure and to its assuming the heteropycnotic condition, almost indistinguishable from that of the inert regions of chromosomes.

2. The thymo-nucleic acid appears in the chromonemata of a given active section (Fölgens reaction) [see also([4])].

3. The degree of these modifications varies in different nuclei of the larva.

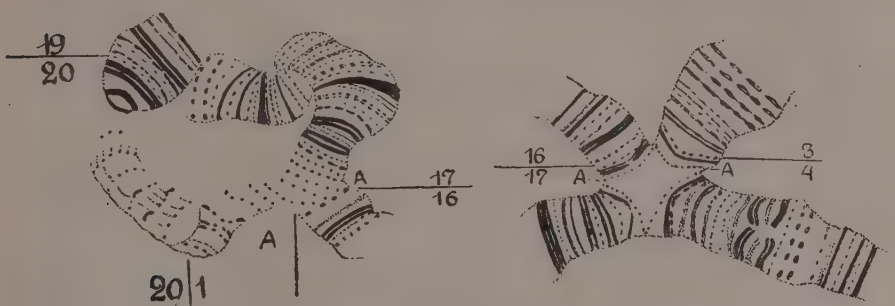
The frequency of occurrence and the degree of expression of the cytological modifications just described run in parallel with the degree of mosaicism in a given strain, i. e. with the relative frequency of mosaic individuals in it and with the size of the area affected in a given individual. Strains in which such modification of some active section are observed prove to be mosaic for the characters depending upon the very same chromosome sections.

Contrary to Schultz's([13]) data, in none of the carefully studied mosaic strains have we found any cytologically discernible deficiency of a section of the chromosome. In the presence of an additional inert region (the *Y*-chromosome) the active section formerly affected returns almost completely to the euchromatic condition. This shows that the chemical processes responsible for the euchromatic or heterochromatic condition of the chromosome sections are reversible.

All the above-mentioned facts justify the suggestion that the course of development of a given character in a mosaic individual depends in a

particular moment not upon the presence or absence of a chromosome section containing definite genes, but upon its definite condition, the latter having a definite morphological and chemical characteristic. The investigation of mosaic strains we have performed has convinced us that the conjugation of chromonemata in a chromosome, the emergence of thymo-nucleic acid in chromonemata, as well as the modification of the character depending in its development upon the chromosome section in question are processes closely connected with one another, as if they were causally related or were all depending upon some more general but as yet unknown factor.

The inert region of the *X*-chromosome of *D. melanogaster*, which has been studied in most detail, is represented in the salivary gland chromosomes by a short chromonema section, having 18—22 chromiols. Along with its property of influencing the structure of active regions, the inert region has another peculiarity consisting in the capacity of its parts to conjugate with one another as well, as with some sections of inert regions of other chromosomes. The capacity of inert regions to conjugate with one another, irres-



a — conjugation of inert regions 1*A*1-3 и 17*A* (*sc*^{8}); *b* — inversion *CBl* — along its boundaries are distributed inert regions in loci 4*A* and 17*A*.

pective of any possible changes in their relative positions as a result of various chromosomal rearrangements, enables the identification of inert regions in any chromosome section, especially in cases when some chromosomal aberration renders the mechanical conditions for the conjugation of the sections in question more favourable. The use of these conjugational properties of inert regions made it possible to establish their presence in various sections of chromosomes.

First of all it proved possible to show that short inert regions are present at the distal ends of all chromosomes of *D. melanogaster* ([3], [11]). Bauer ([1]) has also observed the conjugation of distal ends of chromosomes of various *Drosophila* and *Chironomus* species, which was presumably caused by the mutual attraction of short inert regions present at these ends, but he has given another explanation to these phenomena.

The subsequent finding of heterochromatic regions in plants ([6]) has substantiated our assumption that the presence of inert regions at distal ends of chromosomes is a fundamental feature of the structure of the latter. Further studies have shown that inert regions are present in different interstitial sections of the *X*-chromosome as well ([12]).

New data have been obtained in the course of studying the *sc*^{8} chromosome which carries a long inversion. This inversion brought the proximal sections 15—19 near to the distal inert region, thus making the mechanical conditions for the conjugation of the inert regions they contain with the distal inert

region more favourable. The study of this chromosome has led to the finding of an inert region in the section 17*A* (figure), i. e. just in the region of the right break of the *ClB* inversion, thus substantiating our previous data concerning the existence of an inert region in the section 15*f* (near the forked locus and the B^{M1} and B^{M2} breaks).

The study of the same sc^8 chromosome has led to the finding of an inert region in the 11*CD* section, just in the region of the left break of the δ 49 inversion. The study of the w^{M4} chromosome has shown that an inert region is present in the sections 4*A* (just at the right break of the *ClB* inversion), 7*C*,8*C* and 9*B*.

Thus, the presence of inert regions is now established in the following sections of the *X*-chromosome of *D. melanogaster*: 1*A*1-4, 2*B*, 3*C*3, 3*C*6, 4*A*, 4*C*3-7, 7*C*, 8*C*, 9*B*, 11*AD*, 11*CD*, 12*EF*, 15*F*, 17*A*, 19*E*3-4, 20*AF*.

Comparing these data with what is known about the localisation of a number of breaks in the *X*-chromosome, we can see that many of these breaks have occurred just in the intersticial inert regions or in their immediate proximity.

The coincidence of the points of these breaks with inert regions does not seem to be due to chance. The data presented here justify the suggestion that inert regions play an important rôle in the process of occurrence of chromosome rearrangements. Their rôle in that process may be thought of in two different ways:

1. Chromosomal rearrangements may occur as a consequence of conjugation of two inert regions of the same chromosome (inversions), or of two non-homologous chromosomes (translocations). Many cases of such conjugation of chromosomes in the salivary gland cells of a normal larva have been repeatedly observed [see also McClintock(7)].

2. The sections of the *X*-chromosome in which the intersticial inert regions are located assume the properties of «weak spots», where the chromosome can be bent or brocken most easily. This peculiarity is most sharply expressed in the sections 11*A* and 19*E*. It is probable that the breakability of the chromonema of the inert regions is caused by a conflict of the forces of attraction, but it seems also probable that it depends upon a specific chemical structure of the sections in question. Taking into account the breakability of the chromonema of the inert regions, one can think that such regions may break more frequently irrespective of their conjugation with one another. If this is true, the chromosome rearrangement must be a result of two independent breaks.

It is difficult to say according to which of these two schemes do the chromosome rearrangements actually occur and whether they occur only according to one or both of them, but in any case the important rôle of inert regions seems to be obvious.

The probable existence of inert regions in plant chromosomes as well (inasmuch as heterochromatic sections have been described there), the occurrence of chromosome rearrangements in plants, and the rôle played by chromosome rearrangements in evolution suggest the idea that the inert regions of chromosomes have played a definite rôle in the process of evolution. We are inclined to think that in some cases the peculiarities of chromosome behaviour in plants, usually explained by structural hybridity (ring and chain formation), could be more adequately explained, were the possible rôle of inert regions in the processes involved taken into account.

The last question revealed by the data presented here concerns the relation between the inert regions and the mutability of genes. It must be pointed out that some of the most frequently mutating genes in the *X*-chromosome are located in the immediate proximity of the inert regions. Such are the

y (1 *A*1-4), *w* (3*C*3), *ct* (7*C*), *lz* (8*C*), *f* (15 *F*) and the *bb* (20*C*) genes. This seems to be no mere coincidence. Considering this fact in connexion with the increasing frequency of *X*-ray mutations in chromosome sections approached to the inert regions as a result of chromosome rearrangement ([2], [14]), and with the somatic variability of characters depending upon genes located in such sections [mosaicism, ([13]),([9]) ([10]) and others], we are justified in suggesting that the spontaneous mutability of genes is also somehow related to their position in respect to the inert regions [see also Noujdin([10])].

The list of inert regions of the *X*-chromosome presented in this paper is undoubtedly far from being complete. It seems not exluded that along with the proximal and the distal inert regions they constitute a fundamental feature of the chromosome structure and are nessessary elementary parts of every section of the chromosome.

Noujdin already came to this conclusion on the basis of his genetical data.

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PLANT ANATOMY

A CASE OF FORMATION OF ADVENTITIOUS CAMBIAL RINGS IN THE STEM OF COMMON LIME-TREE

By A. A. RICHTER, Member of the Academy

In Fig. 1, which represents a cross-section of the stem of a common lime (*Tilia cordata* Mill.), two elongated woody formations are to be noticed directly adjoining to the remains of the bark efoliated from the stem, whose main cylinder of wood is seen to have 23 annual layers. This efoliation, combined with dying of half of the wood (marked out by a characteristic boundary, distinctly observable in Fig. 1), is due to the tree having been struck by atmospheric electric discharge; the lightning, through sudden heating and vapour formation, partly destroyed the bark, partly separated it from the wood and left it in the position of a tangent to the circumference of the stem.

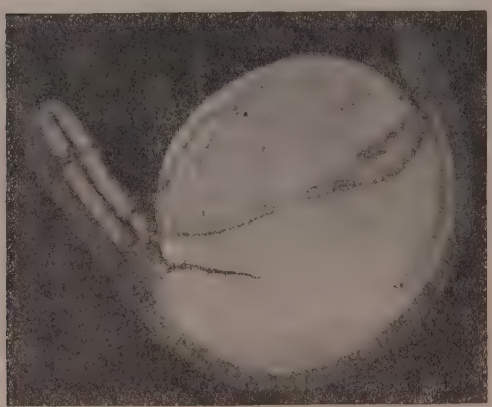

Fig. 1. Cross-section of lime stem (*Tilia cordata* Mill.).

Restoring of the parts injured set in upon traumatism. As may be distinctly seen in the photograph, a ligneous excrescence has started formation under the short piece of bark left, which is usually observed to form in woody plants in the case of their bark being injured. It has a cylindrical shape and is overgrown with cortical tissue, and about it remains of excrescent tissue (callus) are visible. But besides this there are two additional «stemlets» formed directly on the detached and bent parts of the bark. With one side they adjoin to the old bark of the parent stem, while their other side is clothed with their own young bark. The adventitious stemlets include a dark core, as a kind of pith around which three annual layers of wood have been formed by the tissues, and these are clearly seen to bend round the pith at the end of the elongated stemlets.

Anatomical analysis has revealed that the tree surface of the stemlets is covered with a thick layer of cork formed in the thickness of meristematic excrescent tissue (Fig. 2). The meristem gives origin farther inward to liber

tissue which begins with irregularly scattered elements (Fig. 3) and then forms a regular cortical tissue with interlayers of thick-walled liber elements which alternate with areas of thin-walled cells (Fig. 4).

On the opposite side, where the stemlets are linked to the parent bark, no peridermic cells have formed and the cortical tissue of the old stem is seen to pass directly into young liber tissue of the stemlets, being distinguished from the latter by its medullary rays markedly broadening to the periphery. It is separated from new formations by nothing but 2—3 layers of parenchymal cells belonging to what was formerly the excrescent tissue. The central brown layer, which plays the part of the pith, has proved to consist of browned parenchymal cells from the former excrescence. Through several layers of elongated cells with lignified walls it passes into typical cells of limewood, as vessels etc. (Fig. 5).

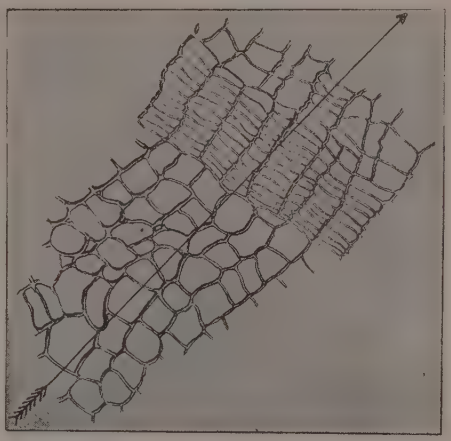

Fig. 2. Outer layers of excrescent tissue (callus). Left, surface of wound; somewhat deeper, thick layer of cork; further inwards, undifferentiated excrescent tissue. Cross-section.

From these anatomical relations we are entitled to conclude upon the origin of the new formations. In course of regeneration out of that portion of the bark tissue detached from the stem which did not lose its vitality, very likely this was cambium, there has been formed a sufficiently thick (up to

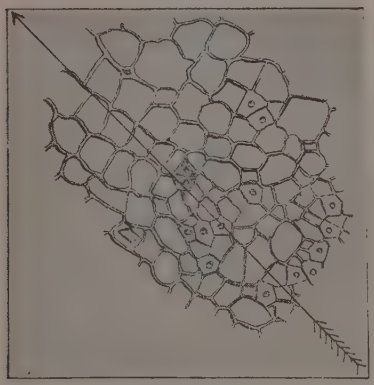

Fig. 3. Outset of differentiation of excrescent tissue. There appear dispersed liber fibres and sieve-tubes. Cross-section.

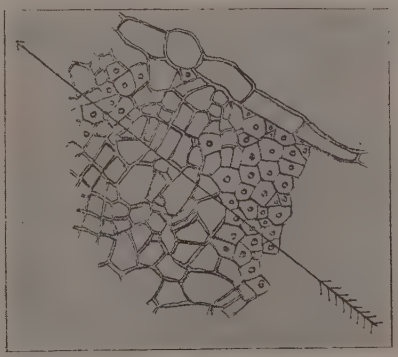

Fig. 4. Normal cortical tissue. Cross-section.

1 mm) layer of excrescent tissue which fenced itself from the free periphery by a layer of cork. In the thickness of the excrescence new cambial layers became then differentiated, but instead of going in allignment with the layers already present, as it usually happens when a very narrow sector of a stem is restored to a full stem ([1]), for instance in grafting, and as it is also the case with

the cylindrical excrescence in our photograph, they formed cambial ellypses enclosing the excrescent tissue («pith») all around and then gave rise to the formation of closed cortical layers and three annual layers of wood.

The formation of «secondary» cambium out of excrescent tissue is a matter of course, as it has been pointed by N. P. Krenke (2). Noteworthy in the case reported is the regular orientation of the secondary cambial ring, forming new independent adventitious stem columns.

One can not help thinking of those peculiar pictures which are presented by «abnormal» structures of the woody stems in dicotyledons when the latter

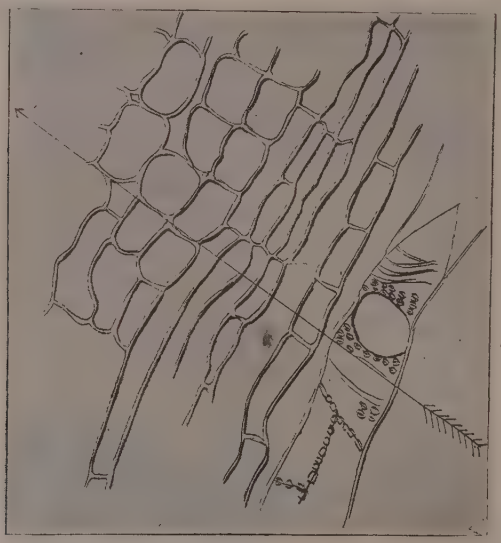

Fig. 5. Transition of «pith» excrescent tissue into wood. Longitudinal section.

(*Serjania*, *Paullinia*) have adventitious stemlets with their own wood and bark arisen as secondary formations in the bark of the parent stem around the central cylinder of wood.

It is a matter for regret that as far as we are aware there are no investigations concerned with the mechanics of the development of those formations, except purely teleological attempts to explain the phenomenon, which consider the transformation of an erect columnar stem into a twisted one. Few have also been the attempts to reveal the causal regularity in regeneration processes, so important the latter are in a number of cases for man's practical activity.

The pictures here described emphasize the immeasurable potentialities concealed in the nature of plant organism. Much is to be expected from a more close study of them, which will afford us means for controlling the development of plants.

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PLANT PHYSIOLOGY

ACTIVITY OF GROWTH PROCESSES—PRINCIPAL FACTOR IN FROST RESISTANCE OF CITRUS PLANTS

By S. M. IVANOV

(Communicated by N. I. Vavilov, Member of the Academy, 25.XII.1938)

It is claimed that the principal factors in the resistance of plants to frost are the pressure in cells of soluble carbohydrates and other osmotic-active substances, on the one hand ([7],[6],[1],[15],[16]), and the high water retaining power of plasmatic colloids, on the other ([1],[10],[9]).

The store of soluble carbohydrates and the water-retaining ability of plasmatic colloids both keep increasing during the hardening process in plants. And, in turn, the frost resistance of various species and varieties is believed to go parallel with their being more or less able to harden.

Since recently the process of hardening has been conceived to comprise two periods. The first goes on in the light at positive temperatures close to 0°C and is determined by storing of soluble carbohydrates; the second proceeds at negative temperatures, which do not damage the plants. Not until they are sufficiently high in sugars can the plants go through the second phase, and the first period is therefore indispensable for the second to take possibly place. Some changes responsible for the resistance of plants to frost have been supposed to occur in the colloids of the plasma owing to the withdrawal of water on slight freezing ([19]).

Though considerable light is thrown on the question, no exhaustive explanation is given by the present notion of causes accountable for the frost resistance of plants. This has been the reason for the failure of numerous attempts to interprete frost resistance in indirect way by the action of some internal factors with which it has been supposed to be linked ([8],[14],[2]). And no adequate means have we therefore at our disposal to induce such frost resistance in plants as would fit the requirements of agricultural practice.

We set before us the task to elaborate methods by whose means the resistance of citrus plants to frost could be increased, and started with the present investigation which had for object to clear up the causes responsible for different resistance of plants to low temperatures.

From our own observations and a number of data reported in literature we are entitled to advance the assumption that the responsible cause of plant's frost resistance lies in the ability of plasmatic cells to stand the coagulating action of low temperature rather than in protective rôle of specific substances, or else in the amount and condition of water in the plant. This conclusion was not reached without supposing that the stability of the plasma

to the harmful influence of frost is inversely dependent upon the intensity of growth and functional activity of cells connected with the latter.

In order to verify this supposition we started experiments with citrus plants in the year 1937—1938. We were endeavouring to clear the dependence of the frost resistance of different species and varieties upon the activity of growth processes and the content of sugars and water in plant. Furthermore, we studied the ability of citrus plants to harden and the character of changes in the condition of plants which had increased their frost resistance on hardening. We also estimated the stability of plasmatic colloids to the coagulating action of low temperature in the sap of the leaves endowed with varying frost resistance.

On account of the space available only part of those experiments—which bear out our chief conclusions—can be reported here.

Frost resistance was estimated by freezing the plants in cooled chambers. The value for the activity of growth processes was the content of glutation in plants. It was the physiological rôle of the glutation which suggested to express in terms of its content both the activity of growth and that of cell function related to the latter. Glutation is known as the natural activator of cell division, differentiation and growth. Its amount is always greater where the processes of growth are more active ([4,5]). Its physiological rôle moreover is that of a regulator of a number of fermentative processes, i. e. the trend of cell metabolism processes is controlled by glutation ([13]).

The amount of glutation was estimated after the method of Woodward and Fry ([12]) which was slightly modified to serve the purpose of plant's test. Only the reduced, more active, form of glutation was considered.

In February we tested the resistance to frost of twigs from mandarin Unshiu which were to different extent advanced in growth. There was also made a parallel estimaton of glutation and sugars in them. Two year and ten year old plants were subjected to test. The activity of growth was higher in the former compared to the latter. The twigs we took from both kinds of plants had shot out in spring, in summer (without resuming growth in autumn) and in autumn. The age of the twigs was closely related to the activity of their growing condition. Glutation was estimated in the liber and leaves of the twigs and plants under comparison. The results obtained have shown the amount of reduced glutation to increase with the activity of growth processes (Table 1).

Table 1

Age of plants	Age of twigs	Reduced glutation in mg per 1 g		% of injured twigs		Amount of carbohydrates in % of absolutely dry weight					
						Twigs			Leaves		
		Liber	Leaves	at—9°C	at—10°C	Monosaccharides	Disaccharides	Total sugars	Monosaccharides	Disaccharides	Total sugars
10 years	Spring growth .	0.1452	0.6819	—	—	—	—	—	—	—	—
	Summer » .	0.1679	0.7666	15	19	2.33	5.27	7.88	3.15	5.67	9.12
	Autumn » .	0.1979	0.9747	65	100	4.47	6.25	11.05	5.69	5.24	11.21
2 years	Spring » .	0.1856	0.7626	—	—	—	—	—	—	—	—
	Summer » .	0.1896	0.7949	50	52	2.03	6.95	9.35	2.05	7.39	9.83
	Autumn » .	0.2421	1.3234	88	100	4.85	6.28	11.47	16.35	6.03	12.70

The twigs from two year old plants contained more glutation than the corresponding twigs from 10 year old plants; the fall-shot twigs more than the summer-shot ones.

The test for the frost resistance of twigs has shown it to be related to the content of glutation (Table 1). Frost resistance was lower in plants with greater amount of glutation which were therefore in more active condition of growth.

No relation between frost resistance and content of sugars could be detected in this experiment.

The experiment of March 4, 1938, on lemon, orange and mandarin Unshiu goes also to confirm these results, since there was noticed an inverse relation between frost resistance and the amount of reduced glutation in various species of citrus plants, while no relation whatever was found between frost resistance and the content of sugars and water (Table 2).

Table 2

Plant	% of injury at—8°C		Reduced glutation in				Total sugars in % of abs. dry weight		% of water in leaves
			Liber		Leaves				
	Twigs	Leaves	mg per 1 g	%	mg per 1 g	%	Twigs	Leaves	
Mandarin Unshiu . .	0	18	0.1574	100	0.6537	100	8.36	12.37	56.6
Orange	8	55	0.1775	113	0.8796	134	8.64	8.73	61.7
Lemon	60	100	0.2179	139	1.1540	176	7.27	10.99	60.8

The most frost resistant is mandarin: at —8°C its twigs escaped any damage and the ratio of leaves perished was 18%. Among orange twigs 8% were injured at the same temperature and the figure of perished leaves amounted to 55%, while 60% and 100% were the respective figures for lemon. Accordingly the least amount of glutation was stated in liber (0.1574 mg) and leaves (0.6537 mg) of the mandarin. In the orange twigs, which were less resistant to frost, the amount of glutation was above that in the mandarin by 13% in liber and by 34% in leaves. The least frost resistant lemon had its amount of glutation in the liber and leaves 39 and 76% higher respectively, as compared to the mandarin.

The content of sugars in the twigs and leaves of the plants compared will not always go parallel with their frost resistance. In the experiment here reported the plants compared differed greatly in resistance to frost. But with less pronounced differences we have also observed the frost resistance to vary inversely with the amount of glutation and, hence, with the activity of growth processes. As to the relation between frost resistance and the amount of sugars it failed to appear in some cases.

The figures in Table 3 give the data of quantitative analyses for glutation and sugars in plants of 3 mandarin varieties, 3 orange varieties and 3 lemon varieties. Frost resistance of these plants is characterized by average results from the tests in December, January and March. The data on frost resistance and content of carbohydrates are quoted from a paper by E. S. Moros and M. V. Kotlyarova ([17]), where the results of the investigation of the above plants are given.

Table 3

Plant	Variety	Temperature of freezing, °C	Injury in %	Reduced glutation in mg per 1 g	Total sugars
Mandarin	Shiva-Mikau	—10.5	35	0.6375	9.89
Unshiu	Unshiu	—10.5	49	0.6940	9.35
	Clementin	—10.5	71	0.7384	6.61
Orange	Ananas	—9.5	48	1.6503	9.00
	Parson-Braun . . .	—9.5	52	1.6987	8.04
	511	—9.5	61	1.7552	7.52
Lemon	Meyer	—7	23	1.9650	10.55
	Chakva	—7	36	2.1587	9.48
	Kuzner	—7	41	2.3968	10.65

We have established by a number af experiments that citrus plants are as able to harden as other plants are, although it is believed that subtropical plants are lacking in this capacity ([11,18]). The plants increase their frost resistance considerably through being subjected for several nights ([3–8]) to the action of positive temperatures close to 0° C and to negative ones provided the latter are not injurious. Inquiring into the change in the condition of plants induced by this treatment, we have been successful to state that the increase in the resistance to frost is virtually linked with the decrease in the amount of reduced glutation. The content of water in the leaves, the amount of sugars, the activity of catalase and the coefficient of temperature (Q_{10}) proved to be not definitely related to frost resistance. Accordingly, in the process of hardening too, the increase in the resistance to frost is related to the depression of growth processes or, strictly speaking, to the fact that the functional activity of the cells, which is linked with the processes of growth, is abated. This is exemplified in Table 4 by the results of an experiment on hardening lemon plants at positive temperatures. The same results have been obtained with negative temperatures when the first phase of hardening varied in length.

Table 4

Conditions of hardening	Injury at—6°C		Reduced glutation		Total sugars	Q_{10}	% of water in leaves
	% of dead twigs	% of dead leaves	In mg per 1 g of leaves	%			
Not hardened	86	100	1.2346	100	9.30	2.14	63.2
at—6°C for 8 nights . .	32	98	1.1984	96	8.21	2.21	62.1
at+2°C for 8 nights . .	6	70	0.8402	68	12.03	2.36	63.4

From the similar nature of changes in the condition of plants both at the first and the second stage of hardening we have to conclude that there is no good reason for dividing the process of hardening into two essentially different periods, even if they are considered as correlated ones. It seems to be virtually one process, which consists in hardening the functional

activity of the cells and is related to growth, refusing any distinction except a quantitative one, and this is determined by whether positive or negative temperatures prevail on hardening.

To verify the truth of the supposition that the changing activity of growth affects the stability of plasma, we carried out experiments in which we determined the stability of colloids in the juice expressed from ground leaves varying as regards the activity of growth processes and taken from both hardened and not hardened plants. In order to obviate the influence the differences in the content of protective substances would exert upon the stability of colloids, we took care to change the osmotic properties of the juice in the samples to be compared by grinding the leaves to different extent before the juice was expressed from them, by drying and diluting the juice and by adding osmotic-active substances to the latter. The method for determining the stability of colloids was the freezing of the juice and the action of electrolytes.

It has been established that the leaves with higher activity of growth processes give a juice whose colloids are by far less stable than in leaves with weakened growth. Comparing, for instance, the leaves of pompelmous from the summer and autumn growth we found the amount of glutation in the former to be 0.7868 mg per 1 g as against 1.7471 mg in the latter. Freezing the juice of those leaves at —6°0 we noticed that, whatever the concentration of osmotic-active substances, the colloids of the juice expressed from the summer leaves suffered no appreciable changes, while those in the juice of autumn leaves precipitated almost completely.

Following conclusions are to be drawn from the results obtained:

1. The principal factor in the resistance of citrus plants to frost is the ability of the plasma to withstand the coagulating influence of low temperature.

2. The stability of the plasma depends inversely upon the activity of growth and its connected functional activity of the cells.

3. As the amount of reduced glutation characterizes the activity of growth processes, it can serve as indirect indicator of the relative frost resistance of plants.

It is very likely that the dependence we have revealed between frost resistance of citrus plants and the activity of growth processes in them is valid with regard to other plants too.

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PLANT PHYSIOLOGY

RELATIONSHIP BETWEEN DISTRIBUTION OF VARIOUS WHEAT FORMS AND THEIR RESISTANCE TO ATTACKS OF FRIT-FLY (*OSCINELLA FRIT* L.)

By P. G. CHESNOKOV

(Communicated by N. I. Vavilov, Member of the Academy, 26.XII.1938)

The question of how the geographic distribution of crops is related to their immunity against affection by insects has but slightly been broached by the investigation. Most of the few data available on the subject refer to the immunity of grape against *Phylloxera vastatrix* Planch, and part of them to the immunity of apple-tree against *Eryosoma lanigerum* Hausm.

With regard to other species no data are available in the literature except for some isolated facts being reported which, in our opinion, betray the existence of regularities in the geographic distribution of immunity and resistance against some or other insect, but the authors failed to bring this into evidence, owing possibly to the variety of forms in the plants under their investigation.

While working for these three latter years in Pushkin laboratory of the All-Union Institute of Plant Industry on the characterization of world's wealth of spring wheats respecting the resistance to frit-fly attacks, we, as early as in 1935, established, among the species whose area of distribution is very wide (*Tr. durum* and *Tr. vulgare*), the presence of separate groups characterized whether by a definite resistance or susceptibility to the attacks of frit-fly larvae. Thereby an unmistakable relation between their origin and susceptibility came to light. This was, however, conflicting with the statements advanced in a number of works formerly published and compelled as to verification, which involved a great variety ot wheats (450 forms) in the study. In course of this work and in treating the data obtained we availed ourselves of the materials on agro-ecological classification of wheats elaborated under the guidance and direct assistance of N. I. Vavilov, Member of the Academy.

The conclusion to be drawn from our investigation is that definite regularities are observed among various species and forms of spring wheat in the geographic distribution of their susceptibility by the frit-fly. These regularities are as follows:

1. Forms of the wheat species characterized by a small area of distribution differ but slightly in their resistance to the insect. In the widespread species (*Tr. durum*, *Tr. vulgare*), on the contrary, the range of variation in the resistance of different forms is extremely broad. This is illustrated by Table 1.

Table 1

Species	Area of distribution	Number of forms investigated	Injury	
			from	to
Tr. monococcum .	Caucasus, Balkan peninsula, Spain, Morocco, Jougo-Slavia (mountains), Germany (mountains)	21	1.8	23.1
Tr. persicum . . .	Armenia, Georgia, Daghestan (mountains)	9	2.7	23.2
Tr. Timopheevi .	Western Georgia	6	3.3	13.5
Tr. sphaerococcum.	India, Punjab	6	59.0	65.0
Tr. dicoccum . . .	Abissinia, Morocco, Aravia, India, Transcaucasus, Volga provinces, North America	37	0.9	34.5
Tr. durum	Mediterranean countries, Asia Minor, Steppe regions in the European part of the USSR, Siberia, Middle Asia, Abissinia, North America, Argentina, Australia	135	5.1	85.6
Tr. vulgare . . .	Wherever wheat is cultivated in Old and New World	250	3.6	90.0

2. Wheat forms from a given locality exhibit the greater differences in their resistance to the insect concerned, the more diversified is the character of agro-ecological conditions prevailing there. By way of example we can refer to the hard wheats originating from such a small territory as isl. Cyprus is. Striking differences in the susceptibility are observed among them, while in the vast steppe region of the USSR most of the forms of this species are distinguished by nearly the same degree of resistance. So again the soft wheats from Armenian and Georgian Soviet Socialist Republics show a much wider range of variations respecting the susceptibility of separate forms as compared to the soft wheats from the forest-steppe and steppe zone in European Part of the USSR and Siberia, though the latter are spread over by far more extensive territory.

3. Within separate species the distribution of forms according to their susceptibility to frit-fly's attacks is related to the biological peculiarities these forms have acquired through living in a definite medium (origin) or by dint of selection (whereby account was taken of the peculiar character of the environment in the respective zone). Table 2 is an illustration of the geographic distribution of susceptibility of wheats according to agro-ecological classification. The data refer to the year 1938.

The results of repeatedly sowing wheats for three years under highly infecting conditions was that most of the soft wheat forms of Indian, Chinese, Middle-Asian and Pamir-Badakhshan groups had the main stems strongly injured on sprouting and at the outset of tillering, while the Boreal and Forest, Forest-Steppe, East-Siberian groups proved to be rather slightly affected. Mild injury was observed in most of the steppe group forms (standard varieties of the steppe zone of the USSR, obtained through individual selection) and in part of the old aboriginal varieties of the West-European group. Some rather slightly susceptible forms were found among Mediterranean and Caucasian wheats, though both groups, as a whole, should be properly classified with moderately susceptible forms. To moderately and strongly susceptible forms refer the aerophilous types of Armenia, Inner Anatolia and Syria.

Among groups consisting of hybrid varieties the Canadian group is especially worth mentioning on account of a rather low susceptibility of the bulk of its forms. Produced by crossing of the Indian ecotype with West-European and Siberian ones, Canadian varieties have inherited the resistance of the latter to the attacks of frit-fly. Unfortunately, limitations of space forbid any extended consideration of a number of other groups within the species above-mentioned, as well as of the other species of wheats. We should like to point out only that the species *Tr. compactum* reveals the same picture in the distribution of its forms with reference to the susceptibility as is presented by the genus *Tr. vulgare*, and this goes to confirm the regularities we have outlined here. Mention needs also to be made of the species *Tr. sphaerococcum* which is analogous to the species *Tr. vulgare* in including strongly susceptible forms within the respective agro-ecological group (from Punjab).

From the data at hand we can conclude that those soft wheats which adapted themselves to the conditions prevailing in the forest and forest-steppe zone of Europe, Siberia and North America are less liable to attacks of the frit-fly, compared to the xerophilous type of soft wheats, which is affected much more strongly and is not resistant to the injury.

This is exemplified most conspicuously by a number of strongly susceptible forms of Bogar wheats from Middle Asia and by xerophilous forms of the Armenian-Turkish group (type Galgalos) of Syrian soft wheats and part of the soft varieties from Asia Minor (Anatolian group). Strong susceptibility too was observed in *Tr. dicoccoides* and *Tr. vavilovianum* which refer to xerophylous types.

Within the species *Tr. durum*, the Mediterranean, Anatolian and, partly, Syrian (Khoranki) groups have their main stems rather slightly susceptible and prove highly resistant when injured. In the latter of these groups considerable susceptibility was recorded under the conditions of North Caucasus (Voroshilovsk, Zapadnoe Polye, 1938), while in the north (Leningrad province, Pushkin) the plants were affected but slightly when growing there for 4 years. Hard wheats from Abissinia and from the steppe zone of the USSR are antipodes of the Mediterranean group. Their high susceptibility seems to be linked with the weakness of their stem tissue, while their poor resistance when injured may be explained by reduced tillering power.

The relation between resistance of hard wheats and their hydrophility is not so pronounced as to be admitted without further investigation. Strong differences in the susceptibility between the Mediterranean and the Steppe forms plead, however, in favour of such a relation.

4. Data accumulated within these latter four years furnish reliable evidence that the ecotype the given form belongs in essentially determines its degree of affection by the frit-fly, as well as the degree of its resistance in case of injury. It is of interest to note here the rather close alliance, as regards the response to the affection by frit-fly, between the most of the forms of *Tr. durum* from Abissinia, which are distinguished by exceptionally wide range of variation of morphological characters, although they belong to the same ecotype. Similar picture is observed in the soft wheats from China. On the other hand, most different gradations of susceptibility are encountered within the same botanical variety when it presents forms of various ecotypes (for instance, *ferrugineum*—including both slightly susceptible East-Siberian and American forms, and strongly susceptible forms from India, Iran and Turkestan).

5. Within the period of 4 years slight fluctuations were observed across years in the susceptibility of separate forms when sown lately in autumn at Pushkin (10 days later against the norm in order to ensure maximum infection

Table 2

Agro-Ecological Groups of Wheat with Reference to Their Susceptibility by Frit-Fly

Agro-ecological group	Number of forms studied	Number of samples injured in %											
		0—5	5—15	15—25	25—35	35—45	45—55	55—65	65—75	75—85	85—95	95—100	$M \pm m$
Tr. vulgare													
Boreal and forest	6	—	2	4	—	—	—	—	—	—	—	—	16.7±2.16
Forest-steppe	10	—	3	7	—	—	—	—	—	—	—	—	17.0±1.4
East-Siberian	8	—	1	5	2	—	—	—	—	—	—	—	21.2±1.85
West-European (aborigenes)	5	—	—	3	1	1	—	—	—	—	—	—	26.0±4.0
Steppe	8	—	—	6	1	1	—	—	—	—	—	—	23.7±2.81
Mediterranean	17	—	—	3	8	6	—	—	—	—	—	—	31.7±1.38
Mountain-Caucasus	6	—	—	2	4	—	—	—	—	—	—	—	26.6±2.16
Armenian-Turkish and Anatolia-Syrian	10	—	—	—	1	4	3	2	—	—	—	—	46.0±2.90
Middle-Asian and Pamir-Badakhshan	11	—	—	—	1	2	2	3	2	1	—	—	55.4±4.54
Chinese-Japanese	18	—	—	—	—	5	4	4	2	3	—	—	53.6±4.11
Indian	9	—	—	—	1	1	2	3	1	1	—	—	55.5±5.06
Canadian (hybrids)	15	—	6	7	1	1	—	—	—	—	—	—	18.0±1.95
Tr. durum													
Inner Anatolian and Syrian	10	—	4	5	1	—	—	—	—	—	—	—	17.0±1.54
Mediterranean	20	—	2	11	7	—	—	—	—	—	—	—	22.5±1.33
Spanish (from mountain localities)	7	—	1	1	4	1	—	—	—	—	—	—	27.1±3.55
Steppe (drought zone of the USSR)	13	—	—	—	2	5	4	1	1	—	—	—	45.4±2.91
Abissinian	8	—	—	—	—	—	—	2	2	3	1	—	73.7±3.56

by the fly). Yet, this did not change the relations in which different forms stand to one another with regard to susceptibility. In none of the years have we recorded that the forms from India, Abissinia and Iran were less injured than or even approached to the slightly susceptible forms of the forest and forest-steppe zone of the USSR, Canada and USA. Hence, within one point of cultivation the range of variation of susceptibility of a single form across the years is rather narrow, provided the conditions of cultivation do not greatly deviate from the norm. The principal factor in these fluctuations is the number of insects and the sexual activity of the latter during the period when the plants are most liable to infection.

Yet, with sharply varying environmental conditions (far outside the normal conditions prevailing in the given locality) we obtained remarkable changes in the susceptibility within the same forms. Thus, for instance, some forms from Eastern Siberia slightly susceptible when sown lately in autumn proved to be rather badly attacked after being sown lately in summer (after July 25, abnormal time), especially when suffering excessive humidity during the latter half of the summer, as it was the case at Pushkin in the year 1935. On the other hand, certain forms from Afghanistan reduced their susceptibility under the above conditions. It seems therefore very probable that different response would be observed in some form to the infection by frit-fly, if we were to test it in sharply different points of this country. This result might be favoured also by the possible differences in the specialization of the frit-fly according to the conditions in the respective zones of the country. The presence of certain differences in the susceptibility of separate forms and groups was revealed when comparing the data obtained in the year 1938 at Pushkin (Leningrad province) and at Voroshilovsk (North Caucasus). Thus, for instance, hard wheats (*Tr. durum*) from Syria slightly susceptible in the former locality turned out to be strongly affected in the latter. On the other hand, the hard wheats from the steppe zone were injured at Voroshilovsk by far less than at Pushkin. Between hard wheats from Abissinia, soft wheats from India, China, Asia Minor, Syria, Eastern Siberia, the forest and forest-steppe zones of the USSR, Canada and a number of others, the relations of susceptibility were the same for both of the above localities.

6. Inquiring into the regularities in the resistance of crops to the attack of frit-fly with reference to geographic distribution of crops and checking the data obtained in such zones of this country, as are most different with regard to agro-ecological conditions, we will accumulate abundant material, from which conclusions of great practical importance can be drawn, and this will enable us to detect the responsible factors of resistance and to trace the ways to a proper selection of components, when crossing wheats to be cultivated in regions where they frequently suffer the harmful activity of the frit-fly.

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PALEONTOLOGY

DISCOVERY OF A LOWER DEVONIAN FISH FAUNA IN THE USSR

By D. OBRUCHEV

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During the last few years earliest vertebrates (fishes and agnathi) have been obtained in the Union from a number of new localities, but practically all of these localities refer to the Upper Devonian or Upper Middle Devonian. Only two localities are known in older beds—in presumably Lower Devonian beds of the Kureyka, a tributary of the Yenissey (1), and in the Upper Silurian of the Belaya River, Urals (2). In 1937, remains of fossil fishes were found in two places—on Taimyr peninsula and near the Lake Balkhash. These fossils can be safely synchronized with Lower Devonian marine fish fauna of other countries. These discoveries allow us also to revise the age of the fish-bearing beds on Kosva and Vilva rivers, Urals, referred to the upper half of the Middle Devonian (2).

On the Taimyr the fossils were obtained by N. P. Anikeev and A. I. Gusev, of the Arctic Institute, on the right bank of Tareya, a tributary of Pyasina, from a series bearing a rich fauna of corals, brachiopods, ostracods, trilobites and cephalopods. Fish remains occur in a bed which includes no other fossils. Brachiopods, corals and trilobites from a bed overlying the fish-bearing one have been studied by O. I. Nikiforova, B. B. Chernishev and N. E. Chernisheva. They consider this fauna to be of Lower Devonian age, as there are no distinctly Silurian forms, while new species resembling the Devonian forms are numerous. The fish-bearing bed is underlain by limestones still containing Silurian forms, besides the presumably Devonian ones. Thus, there is a degree of uncertainty in the age definition basing on invertebrates. The fish remains belong mainly to two new species of *Lunaspis*; besides there are a scale of *Porolepis* n. sp., unidentifiable remains of *Arthrodira*, and an *Onychodus*-like tooth.

The second finding comes from Aynabulak, north of Lake Balkhash, and consists of 2 specimens of a new species *Lunaspis*. It was transmitted to me by N. L. Bublichenko and M. F. Neuburg from the collection of Loginov (the Kazakh Branch of the Academy of Sciences). Here the fish remains are associated with pelecypods, bryozoans, and plant remains. The identification of brachiopods from the same outcrop made Bublichenko to define the age of the beds as D_1^2.

Fish remains from these two localities are Lower Devonian. The genus *Porolepis* is known in situ from the Upper Siegenian and Lower Coblentzian of Rhineland (4) and from the Wood-Bay series (lower part of the Lower Devonian) of Spitzbergen(3). It has first been described from a boulder, which is generally claimed to belong to Lower Devonian. There are 3 species of *Lunaspis* known from the Rhenish Lower Devonian: *L. heroldi* Broili, from

the Hunsrückshiefer; *L. broilii* Gross, from the Upper Siegenian up to Upper Coblentzian; *L. prümiensis* (Kayser) from the Upper Coblentzian(4). A fourth species, *L. arctica* (Heintz), comes from the upper part of the Grey-Hoek series (upper part of the Lower Devonian) of Spitzbergen (3).

Porolepis from Taimyr differs from all the known species by the presence, on overlapped areas, of flat tubercles, like those on the scales of *Glyptolepis*.

Lunaspis sp. 1 from Taimyr (the majority of remains) in its body-plates bearing scarce concentric ridges is nearest to *L. heroldi*, but has spinalia with spines on both anterior and posterior borders, as in *L. arctica*. *Lunaspis* sp. 2, a small species with delicate and narrowly set concentric ridges, is somewhat like *L. broilii*. *Lunaspis* from Lake Balkhash is larger than the two Taimyr species. Its anterior ventro-lateral resembles that of a specimen which has not without qualification been referred to *L. arctica*[(3), Pl. 1, Fig. 5], except for that it is less bevelled, the angle between the lateral and the anterior borders being not much wider than a right one. The centro-nuchal bears more pronounced tubercles than in other species.

In applying Wedekind's (5) tripartite division of the Rhenish Lower Devonian, we can refer the Taimyr fish-bearing beds to the middle part (Lower Coblentzian) or to the uppermost layers of the lower part (Siegenian), while on the Balkhash we are dealing with the upper part (Upper Coblentzian). Evidence: the presence on Taimyr of *Porolepis* and of smaller forms of *Lunaspis*, more like earlier species; on Balkhash, the absence of *Porolepis*, and the *Lunaspis*-form being larger and more like the younger species, *L. arctica*. The brachiopods from the latter locality point also at D_1^2.

In a paper, recently published but written in 1934 (2), I described under the names of *Actinolepis tuberculata* Agass. and *Porolepis uralensis* sp. n. fish remains from the rivers Kosva and Vilva, Urals, coming, on field evidence, from the Middle Devonian. Now, when *Lunaspis* is adequately described and depicted (3,4), and when we dispose of specimens of this genus from Taimyr and Balkhash, we can see that what was described as *Actinolepis* is in reality either an indisputable *Lunaspis* (the specimen from Kosva, Pl. 2, Fig. 3), or possibly another, undeterminable genus of *Arthrodira*, most likely belonging to *Acanthaspidae* (specimens from Vilva, Pl. 2, Fig. 1). On account of bad preservation of specimens from Vilva no use can be made of the *Acanthaspidae* as the indicators of age, since they were living both in the Lower and Middle Devonian. *Porolepis*, as has been mentioned, is known in situ only from the two lower parts of the Lower Devonian, although in the Posen boulder, as described by Kade, it is associated with possibly Middle Devonian forms [(2), p. 41]; but Kade's drawings are insufficient. So it is more correct to consider *Porolepis* to be an early Lower Devonian genus. Being associated with *Lunaspis*, it points decidedly at a Lower Devonian age. We may thus take it for granted, that the fish beds on Kosva and Vilva, which in my paper were referred to the upper half of the Middle Devonian, belong in reality to the middle layers of the Lower Devonian, and are near in age to the fish horizon of Taimyr. In Urals we have thus a third locality of the Lower Devonian marine fish fauna in the USSR.

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